Proceedings of annual meeting on

Theoretical aspects of Controlled Thermonuclear Research

Held at Ramada Inn — Rosslyn
1900 North Fort Myer Drive, Arlington, Virginia 22209

April 7-9, 1975 Sponsored by the

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NAVAL RESEARCH LABORATORY
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PROCEEDINGS OF THE ANNUAL MEETING ON THEORETICAL ASPECTS OF CONTROLLED THERMONUCLEAR RESEARCH

HELD AT

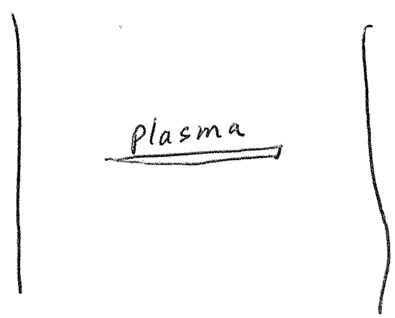
RAMADA INN-ROSSLYN
1900 NORTH FORT NYER DRIVE
ARLINGTON, VIRGINIA 22209

April 7-9, 1975

$\nu' = \frac{\Theta v_T \tau}{r}$

$\left(\frac{r}{R}\right)^{3/2} < \nu < 1$

$n(v_z)_2 = n\left(\frac{T}{2\pi m_e}\right)^{1/2} \exp(e\Phi/T)$



$\phi'' = -\frac{eB^2}{m_+ c^2}\left(1 - \frac{Q_e}{Q_+}\right)$

$V \sim \frac{1}{3} C_s$ on FMI

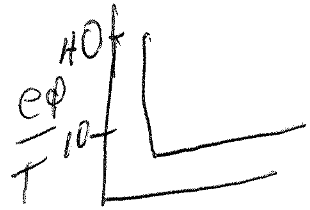
$Q_+ = \int_0^L n v_x \, dz$

$T_\perp = 0 = B_y$

Braginskii Eqns.

$Q_e = 0 \Rightarrow E \simeq \text{const}$

$5 < \frac{\phi}{T_e} < 30$



SCHEDULE

1975 ANNUAL MEETING ON THEORETICAL ASPECTS OF CTR

SUNDAY, 6 April — Registration at Ramada Inn (Main Lobby) (6:00 PM - 8:00 PM)

MONDAY, 7 April

9:00 AM	WELCOME ADDRESS - Dr. Alan Berman, Director of Research, Naval Research Laboratory
9:05 AM	Morning Session Begins (Rosslyn AB)
10:25 AM	Coffee Break (Promenade)
10:40 AM	Meeting Reconvenes (Rosslyn AB)
12:15	Lunch
1:30 PM	Afternoon Session Begins (Rosslyn AB)
3:20 PM	Coffee Break (Promenade)
3:35 PM	Meeting Reconvenes (Rosslyn AB)
4:45 PM	Afternoon Session Adjoins
5:00 PM	Supplementary Session Begins (Rosslyn AB)
6:30 PM	Supplementary Session Adjoins
6:45 PM	Cocktails (Cash Bar) (Shenandoah ABC)
8:00 PM	Buffet Dinner

TUESDAY, 8 April

9:00 AM	Morning Session Begins (Rosslyn AB)
10:20 AM	Coffee Break (Promenade)
10:35 AM	Meeting Reconvenes (Rosslyn AB)
12:15	Lunch
1:30 PM	Afternoon Session Begins (Rosslyn AB)
3:20 PM	Coffee Break (Promenade)
3:35 PM	Meeting Reconvenes (Rosslyn AB)
4:45 PM	Afternoon Session Adjoins
5:00 PM	Supplementary Session Begins (Rosslyn AB)
6:30 PM	Supplementary Session Adjoins
8:00 PM	Executive Committee Meeting (Dogwood A)

WEDNESDAY, 9 April

9:00 AM	Morning Session Begins (Rosslyn AB)
10:20 AM	Coffee Break (Promenade)
10:35 AM	Meeting Reconvenes (Rosslyn AB)
12:15	Lunch
1:30 PM	Afternoon Session Begins (Rosslyn AB)
3:20 PM	Coffee Break (Promenade)
3:35 PM	Meeting Reconvenes (Rosslyn AB)
4:45 PM	Afternoon Session Adjoins
5:00 PM	Supplementary Session Begins (Rosslyn AB)
6:30 PM	Meeting Adjoins

PROGRAM

1975 ANNUAL MEETING ON THEORETICAL ASPECTS OF CTR

MONDAY, 7 April

9:00 AM — WELCOME ADDRESS - Dr. Alan Berman, Director of Research, Naval Research Laboratory

SESSION A - TOKAMAKS, Chairman - H.P. Furth

A1 9:05 AM — J. Tachon
Recent Results of TFR

A2 10:00 AM — M.N. Rosenbluth, D.A. Monticello, A. Simon and B.V. Waddell and R.B. White
Analytic and Computational Aspects of the Reduced MHD Equations

A3 — H. Grad, P.N. Hu and D.C. Stevens
Adiabatic and Resistive Island Behavior of Doublet and Tokamak

A4 — G. Bateman, R.A. Dory, H.R. Hicks and J.W. Wooten
3-D Nonlinear Evolution of Internal MHD Instabilities

A5 — D.C. Barnes and J.U. Brackbill
Toroidal MHD Computations

A6 — A.H. Glasser
Resistive Instabilities in a Tokamak

SESSION B - TOKAMAK AND MHD THEORY, Chairman - C.W. Horton

B1 1:30 PM — W. Grossmann, J.A. Tataronis and H. Weitzner
Stability of the Strongly Non-Circular High-Beta Tokamak

B2 — J.P. Freidberg and J.P. Goedbloed
Equilibrium and Stability of High-Beta Diffuse Tokamaks

B3 — M.S. Chance, J.M. Greene, R.C. Grimm and J.L. Johnson
Numerical Studies of Continuum Modes in Ideal MHD

B4 — A.H. Boozer
An MHD Theory of a Divertor

B5 — J.M. Rawls, D. Dobrott and R.L. Miller
A Transport Model with Moving Non-circular Flux Surfaces

B6 — R.V. Lovelace
Low-Frequency Stability of High-Current Particle Rings

B7 — S. Ortolani
Minimum Energy Reverse Field Configurations

SUPPLEMENTARY SESSION - TWO COMPONENT SYSTEMS AND TOKAMAK MODELING
Chairman - J. M. Dawson

S1 5:00 PM — R.M. Kulsrud and D.L. Jassby
The Neutralized Colliding Beam Torus

S2 — P.H. Rutherford, H.P. Furth, F.G.P. Seidl and D.F. Duchs
Transport Computations for Tokamaks Including Impurity Effects

S3 — Research Staff, Laboratory for Applied Plasma Studies, SAI, LaJolla
Recent Results from Tokamak Modeling

S4 — J.H. Gardner and J.P. Boris
Quasi-Static 2D Model for the Evolution of a General Tokamak Cross-Section

S5 — J. Killeen, K.D. Marx, A.A. Mirin and M.E. Rensink
Energy Multiplication Studies for Two Component Torus

TUESDAY, 8 April - SESSION C - HIGH-BETA PINCHES, Chairman - H. Weitzner

C1 9:00 AM F. Ribe
Review of Toroidal High-Beta Systems

C2 9:55 AM A.G. Sgro and C.W. Nielson
Hybrid Model Studies of Magnetic Field Diffusion Studies

C3 R.C. Davidson, B.H. Hui, P.C. Liewer, S. Hamasaki, M.Z. Caponi, D.B. Batchelor, D. D'Ippolito and J.M. Ogden
Kinetic and Fluid-Numerical Studies of High-Beta Plasmas

C4 H.C. Lui, M.F. Reusch and R. Taussig
Numerical Studies of Initial Dynamics, Equilibrium and Heating of Belt Pinches

C5 R.C. Davidson, R. Gerwin and N.T. Gladd
Anomalous Transport Properties Characteristic of the Post-Implosion Phase of High-Density Pinch Experiments

C6 A. Kadish
Plasma Rotation in a Straight Theta-Pinch

SESSION D - PLASMA HEATING, TURBULENCE AND NONLINEAR THEORY
Chairman - A. Bers

D1 1:30 PM R.L. Berger, R.M. Kulsrud and F.W. Perkins
Plasma Heating Theory

D2 J. Dawson and T. Kamimura
Discussions of UCLA Simulation Program

D3 L. Chen and H. Okuda
Parametric Instabilities and Anomalous Diffusion Near the Upper-Hybrid Frequency

D4 K. Nishikawa, P.F. Kaw, K. Mima, H. Hojo, A. Ishida, H. Tabuchi, Y. Yoshida, A. Hasegawa and H. Ikezi
Envelope Solitons and Envelope Shocks

D5 G. Schmidt
Instability of Envelope Solitons

D6 J.F. Drake, D.D. Fried and Y.C. Lee
Application of Secular Mode Coupling Theory to the Electron Beam Plasma Instabilities

D7 C.E. Seyler, Jr., Yehuda Salu, D. Montgomery and G. Knorr
Wavelength Distributions in Guiding - Center Electrostatic Turbulence

SUPPLEMENTARY SESSION - EXPLORATORY CONCEPTS, Chairman - N. Rostoker

S1 5:00 PM J.R. Freeman, M.M. Widner, J.W. Poukey, T.P. Wright, G.R. Hadley, K.D. Bergeron and J. Chang
Electron Beam Fusion

S2 G.J. Morales and Y.C. Lee
Spiky Turbulence: Its Generation and Effect on Plasmas

S3 P.J. Turchi and D.L. Book
Use of Imploding Liners in Strong Adiabatic Compression of Beam-Target Fusion Plasmas

S4 C.W. Hartman, D.Y. Cheng, J. Eddleman, R. Munger and J. Shearer
Numerical Modeling of the Gas Embedded Z-Pinch

TUESDAY, 8 April (Supplementary Session Continued)

S5 J.R. Hiskes
A Mechanism for Negative Ion Formation in the Surface-Plasma Negative Hydrogen Ion Source

S6 B.N. Moore and W.E. Drummond
Investigation of a P-B^{11} Fuel Cycle for an Advanced CTR Utilizing Magnetic Confinement

WEDNESDAY, 9 April - SESSION E - MIRROR SYSTEMS, Chairman - G. Guest

E1 9:00 AM T.K. Fowler
Review of Open-Ended Mirror Systems

E2 9:55 AM B. McNamara, L.S. Hall, J.K. Boyd and C.A. Wilgus
Three-Dimensional Flux Tube Equilibria in 2XIIB

E3 C.L. Hedrick and L.W. Owen
Single Particle Drift Orbits in a Finite Beta Scalar Pressure Equilibrium for EBT

E4 D.B. Nelson and C.L. Hedrick
MHD Stability and Beta Limitations of the Elmo Bumpy Torus

E5 H.L. Berk and J.J. Stewart, Jr.
Quasilinear Computation of the Double Hump Instability

E6 J.A. Byers, J. Killeen, A.A. Mirin, M.E. Rensink and D.R. Schnabel
Computational Results for Mirror Systems

SESSION F - TRAPPED PARTICLES IN TOROIDAL SYSTEMS, Chairman - H. Be

F1 1:30 PM R.D. Hazeltine and A.A. Ware
New Neoclassical Effects Resulting from Electrostatic Trapping

F2 N. Winsor, B. Coppi and C. Wagner
Resistivity in a Trapped-Electron Plasma

F3 W.M. Tang, J.C. Adam and P.H. Rutherford
Trapped-Particle Instabilities:Destabilization from Curvature Drift Resonances and Stabilization from Inverted Gradient Prof

F4 K.T. Tsang and J.D. Callen
Dissipative Trapped-Electron Instabilities in the Transition from the Banana to Plateau Regime

F5 W.M. Manheimer, J.P. Boris, E. Ott, J.D. Callen and K.R. Chu
Stabilization of the Dissipative Trapped Electron Instability by Mode Coupling

F6 C.O. Beasley, J.E. McCune, H.K. Meier, W.I. van Rij
Calculation and Analysis of a Collisional Trapped-Particle Equilibrium

F7 W.W. Lee, Y. Matsuda and H. Okuda
Numerical Simulation of Drift Waves and Trapped Electron Instabilities

WEDNESDAY, 9 April - SUPPLEMENTARY SESSION - LOW-FREQUENCY STABILITY
Chairman - E. Ott

S1 5:00 PM F.L. Hinton and D.W. Ross
Effects of Finite Collisionality on the Trapped Electron Instability in Tokamaks

S2 H.C. Chan and D.J. Sigmar
Alfven Wave Turbulence Due to Alpha Particles

S3 B. Coppi, F. Pegoraro and G. Rewoldt
Resonant Ion Heating and Slide-Away Regime in High Temperature Plasmas

S4 P.J. Catto, J.D. Callen, W.M. Tang, C.S. Liu and M.N. Rosenbluth
Finite Beta Effects on Drift-Dissipative Instabilities

S5 S.M. Mahajan and P.H. Rutherford
Equilibrium and Stability of Plasma-Neutral Layer in the Tokamak

CONTENTS

POST DEADLINE ABSTRACTS

INJECTION AND TRAPPING OF INTENSE RELATIVISTIC BEAMS IN TOROIDAL MAGNETIC FIELD GEOMETRIES

V. Bailey
Physics International Company
San Leandro, California 94577

Computational studies of a method for efficient, multi-turn injection and trapping of an intense relativistic electron beam in a toroid are discussed. An injection code, which has been developed to calculate the trajectory of a relativistic beam injected into a toroidal magnetic field geometry both with and without the presence of a pre-existing toroidal plasma current is described. The code includes the effects of the beam self-magnetic field, the toroidal curvature of the poloidal magnetic fields, the induced electric fields and the induced currents in the toroidal plasma. The externally applied magnetic fields consist of a toroidal plus betatron type magnetic field geometry. For the case where there is no pre-existing toroidal plasma current, calculations of beam injection and trapping which employ drift injection and trapping of the beam by either the generation of self-magnetic fields of the beam or energy transfer to the plasma are presented. Injection of the beam in the presence of a pre-existing toroidal plasma current is also considered. The coalescence of the beam with the plasma channel or the loss of the beam from the system is shown to be a function of the direction of the injected current with respect to the plasma current, the poloidal angle (lead or lag) between the beam current and effective image current, and the direction of the toroidal magnetic field. Beam trajectories and coalescence with the plasma are shown for several combinations of these parameters.

TOROIDAL MHD COMPUTATIONS

D. C. Barnes and J. U. Brackbill
Los Alamos Scientific Laboratory, University of California
Los Alamos, NM 87544

A numerical algorithm has been used to solve the three-dimensional, time dependent, nonlinear equations of motion in a toroidal geometry. By artifically removing kinetic energy, equilibria which are accessible from the initial conditions may be found if any such equilibrium exists. Two results are presented to illustrate this technique. In the first, the initial conditions are those for a pure toroidal theta pinch. Since no equilibrium exists, the time dependent code follows the plasma motion to the conducting wall. Trace particles are used to follow the fluid motion.

In the second case, a poloidal field is added to the theta pinch. Now, poloidal flux is trapped between the plasma column and the wall so that an equilibrium is found by the computation. While both of these cases are two dimensional in character, it is clear that the same technique can be used to consider arbitrary three dimensional geometries and that a clear indication is provided when no equilibrium exists.

The numerical algorithm has also been used to study the effect of toroidicity on the linear growth rates of the internal kink mode. Results of a computation in a straight cylindrical geometry are given, in which case the growth rate is very small in agreement with theory. The same configuration is considered in the toroidal case with the flux surfaces adjusted to give a toroidal equilibrium with an aspect ratio of order unity. Comparisons of the growth rate and eigenfunction for the straight and toroidal case are presented.

3-D Nonlinear Evolution of Internal MHD Instabilities*

G. Bateman, R. A. Dory, H. R. Hicks, J. W. Wooten
Oak Ridge National Laboratory, Oak Ridge, Tennessee

Internal MHD instabilities appear to be characterized by vortex cells helically twisted along the plasma column, with radial extent depending on the range of q-values in the plasma.[1] We have shown[2] that their 3-D nonlinear evolution is characterized by convection around essentially fixed vortex cells for weak instabilities in an equilibrium with uniform density profile. We will present evidence that this behavior is altered by relatively mild changes in the equilibrium density profile and by changes in the strength (linear growth rate and radial extent) of the instabilities.

*Research sponsored by Energy Research and Development Administration under contract with Union Carbide Corporation

1 G. Bateman, W. Schneider, W. Grossmann; Nuclear Fusion 14, 669 (1974); W. Schneider, G. Bateman; IAEA 5th Conf. Plasma Phys. Cont. Fusion Res., Tokyo (1974) IAEA-CN-33/A12-1.

2 J. W. Wooten, H. R. Hicks, R. G. Bateman, R. A. Dory; "Preliminary Results of the 3-D Nonlinear Ideal MHD Code" ORNL-TM-4784, (1974).

Abstract Offered for the 1975 Annual Meeting on Theoretical Aspects of Controlled Thermonuclear Research

Calculation and Analysis of a Collisional Trapped-Particle Equilibrium*

C.O. Beasley, J.E. McCune,+ H.K. Meier, W.I. van Rij

Using an analytical/numerical method, we have begun studying collisional equilibria of trapped-particle distributions in tokamaks. In order to calculate these equilibria, we use a modified Laguerre-spherical harmonic representation in velocity space, and semi-cylindrical toroidal coordinates in configuration space to write matrix equations for the time dependence of the amplitudes (in this representation) as determined by the drift-kinetic equation and the full Fokker-Planck collision operator. We begin the calculations at $t = 0$ with a Maxwellian plasma and a radial density gradient (and/or temperature gradient, with or without an induction electric field). We show the buildup of the bootstrap current, trapped density, and radial diffusion with time. We also analyze the velocity-space behavior of the non-Maxwellian part of the distribution function, $f(v, \alpha)$ ($v = \sqrt{E/m}$, $\alpha = \tan^{-1} v_{\parallel}/v_{\perp}$), for various collisionalities.

*Research sponsored by the Energy Research and Development Administration under contract with Union Carbide Corporation.

+Massachusetts Institute of Technology

Plasma Heating Theory*

R. L. Berger, R. M. Kulsrud, and F. W. Perkins

Plasma Physics Laboratory, Princeton University
Princeton, N.J. 08540

This paper reports advances in four areas germane to heating tokamak plasmas: (1) The theory of linear parametric instabilities near the lower-hybrid frequency has been extended to show that decay into electron waves and short-wavelength ion-cyclotron modes has the lowest threshold. (2) For frequencies near the ion-ion hybrid frequency, mode conversion is an important absorption process for magnetosonic waves. (3) Fokker-Planck simulations of second harmonic ion-cyclotron heating show good agreement with the ST experiment. (4) Electromagnetic ion-cyclotron beam instabilities can develop in some TFTR neutral beam heating schemes.

* Work supported by U.S.E.R.D.A. Contract AT(11-1)-3073.

ELECTRON BOUNCE MODES IN MIRROR MACHINES*

H. L. Berk and C. E. Nielsen**

Lawrence Livermore Laboratory, University of California

Livermore, California

Typically in mirror machines such as Baseball, the ion frequency is 2--3 times the electron bounce frequency. As a result electron bounce modes, which are mathematically similar to Bernstein modes, can be excited and be destabilized by the ion loss cone distribution. In order for electron bounce modes to arise it is necessary that the bounce frequency be well defined. This arises if electrons are primarily confined electrostatically in a quadratic well. Spreads in the bounce frequency due to axial magnetic confinement or the nonquadratic nature of the well introduce dissipation which is a stabilizing influence. As a consequence, instability is critically dependent on how the ions spread axially due to Coulomb collisions (a stabilizing tendency) or are confined to the mid-plane because of electron drag and charge exchange (a destabilizing tendency). A WKB calculation of this problem has been performed and stability conditions will be presented. Bounce modes are not likely to occur in high density mirror machines such as 2X since the inductive response of the mode forces the parallel electric field to nearly vanish.

*Work performed under the auspices of the U. S. Energy R. & D. Administration.

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Prepared for publication in the Proceedings of the Annual Meeting on Theoretical Aspects of CTR, Washington, D. C., April 7-9, 1975.

QUASILINEAR COMPUTATION OF THE DOUBLE HUMP INSTABILITY*

H. L. Berk and J. J. Stewart, Jr.

Lawrence Livermore Laboratory, University of California

Livermore, California

Target plasma buildup in mirror machines introduces the possibility of transient double-hump modes which can affect the ultimate plasma formed. To study this problem we have constructed a quasi-linear code to study the kinetic evolution of the hot distribution formed by the beam. We assume that instability at the lower hybrid frequency can arise with growth rates greater than the ion cyclotron frequency and we study the quasi-linear equation for two dimensional isotropic turbulence. It is well known that in two-dimensions an unstable particle distribution will relax to a stable one. However, in our calculation we assume that the hot distribution is formed by a continuous beam, and we introduce sinks at the boundary, to simulate the effect of a loss cone. Some examples of the plasma buildup will be presented, and we will show some examples of plasma relaxation oscillations as the distribution alternately becomes stable and then unstable.

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*Work performed under the auspices of the U. S. Energy R. &. D. Administration.

Prepared for publication in the Proceedings of the of the Annual Meeting on Theoretical Aspects of CTR, Washington, D. C., April 7-9, 1975.

Impurity Transport in Tokamak Plasmas*

H. W. Bloomberg
Science Applications, Inc.

J. P. Boris
Naval Research Laboratory

B. H. Hui
University of Maryland

A cylindrical one dimensional plasma numerical code appropriate for impurity transport in tokamaks is presented. The code is based on the construction of Lagrangian cells which move in the radial direction. The outside grid point of the last Lagrangian cell is constrained to coincide with the wall, so that the method is especially useful for those proposed devices where the plasma is compressed by a moving wall. Dissipative effects, leading to magnetic field diffusion, particle drifts, and entropy changes, are conveniently calculated in the reference frames moving with the Lagrangian cells. Once new values of the magnetic fluxes, masses, and entropies corresponding to each cell are found, a pressure balance consistent with these new values is performed. The resultant cell motion can be identified with the $\underline{E}\times\underline{B}$ drift.

The equations for dissipative and neutral particle effects have been solved in conservative form with the use of the FCT algorithm. The effects of chemistry are being incorporated into the code.

*Work supported in part by ERDA

An MHD Theory of a Divertor*

Allen H. Boozer
Plasma Physics Laboratory, Princeton University
Princeton, N.J. 08540

A simple model of a divertor is studied with the classical two-fluid MHD equations. The ion-temperature is assumed zero while the electron mean free path λ_e is assumed to lie in the range $(m_+/m_e)^{1/2} \gg L/\lambda_e \gg 1$ where L is the divertor length. There are three primary results.

(1) If the main body of the plasma is stationary, the electrons flow to the walls in a narrow region near the plasma, which has a scale of $\rho_e L/\lambda_e$. ρ_e is the electron gyroradius.

(2) The ion outflow occurs on a scale of $\rho_+(T_e)$, the ion gyroradius calculated at the electron temparature. This is also the divertor width and is substantially wider than the simple estimate $(D_\perp L/C_s)^{1/2}$ which gives an approximately width $(\rho_+ \rho_e L/\lambda_e)^{1/2}$.

(3) The ion flow velocity into the electrostatic sheath near the neutralizing walls is $C_s = (T_e/m_+)^{1/2}$. The density drops by a factor of $1/\sqrt{2}$ along the field lines from the center of the divertor to the sheath.

Results (1) and (2) are quite different from simple extensions of one-dimensional discharge theory which has formed the basis for discussions of divertors.

* Work supported by U.S.E.R.D.A. Contract AT(11-1)-3073.

Progress on Developing Numerical Simulation Codes to Study Low Frequency Instabilities

J. P. Boris, W. M. Manheimer, R. N. Sudan
Naval Research Laboratory

C. Wagner
Science Applications Incorporated

The usual particle codes are useful to study instabilities if the total time for saturation is less than about $10^4/\omega_{pe}$ and if the basic wavelengths are less than about $20\lambda_{De}$. Since tokomak related instabilities involve far longer length and times, new simulation tools appear to be essential. Work is under way to develop some of these essential tools and preliminary results of this research will be presented.

At the Naval Research Laboratory, this model-development work has proceeded along several lines concurrently. A quasi-neutral fluid code has been written which neglects electron inertia and treats ion inertia and ion larmor radius as a small perturbation. Thus the Alfven and sound speeds, as well as electron and ion plasma and cyclotron frequencies, do not enter as physical limitations. The natural velocity is the drift velocity and the natural time is the grid cell divided by the drift velocity. Such a fluid model is a natural way to study tokomak-related instabilities. In our preliminary tests we study large amplitude drift waves which are easily destabilized by adding in various transport mechanism

A parallel effort is under way to perfect a particle algorithm in which the electric field is computed from a quasi-neutrality condition rather than Poisson's Equation. The related analytic problem is simple to formulate, and one can show that the electron Debye length and inverse plasma frequencies are not relevant length and time scales. The numerical realization of this model has been sought for years. Current efforts center on a 1D model in which the electric field is chosen implicitly to force the constraint $\underline{\nabla} \cdot \underline{j} = 0$. The particle-pushing algorithm is then modified to ensure $n_e = n_i$. Although success in the multidimensional problem is not assured by our 1D successes to date, this quasi-neutral particle code is a very important milestone if one is to simulate tokomak related instabilities that depend critically on details of the distribution function.

We are also considering various hybrid schemes where one species is fluid, while the other is treated as particles. Since one species is a fluid, one can usually utilize a quasi-neutrality condition rather than Poisson's Equation, and thereby avoid the Debye length scale. For the dissipative trapped electron mode, it appears possible to use fluid ions and particle electrons which follow drift motion perpendicular to the field and their normal particle orbits along the field

*Work Supported by ERDA

ICRH Heating of Plasmas and Associated Parallel Temperature Cooling

J. Busnardo-Neto, J. Dawson, T. Kamimura and A. T. Lin
University of California at Los Angeles

We have investigated the heating of plasma by intense ICRH waves through the use of a one and two halves dimensional computer model. The wave was generated by pumping the plasma for a short time with an electric field of the appropriate frequency, wave number and polarization. Strong heating of the perpendicular ion temperature was observed in agreement with theory. In addition a strong cooling of the parallel ion temperature was observed. Reductions in the parallel ion temperature by up to 60% were observed. This effect can be understood as follows. The ion cyclotron wave has a frequency below the ion cyclotron frequency. In order for an ion to absorb energy it must be moving so as to doppler shift the wave frequency to the ion cyclotron frequency. From a quantum point of view the ion absorbs $\hbar\omega$ energy from the wave and must supply the difference in energy, $\hbar(\omega_c - \omega)$, from its parallel motion. We have developed a theory for this effect which appears to be consistent with the observations. We will also discuss this effect from the thermodynamic point of view. This effect might be exploited to plug mirrors. Ions could be heated so as to make up for their energy loss to cold electrons while at the same time they are kept out of the loss cone. One might then operate a wet wood burner of the type discussed in the abstract "Some New Ideas on Wet Wood Burners". One might also use the effect to reduce axial losses in a long straight machine with multi-mirror end plugging.

Work supported by EPRI # RP270-0-0

SIMULATION OF SELF-CONSISTENT MAGNETIC FIELDS IN 2XIIB*

J. A. Byers, A. A. Mirin and D. R. Schnabel
Lawrence Livermore Laboratory, University of California
Livermore, California

The 600A of neutral beam being implemented on 2XIIB is a sufficiently large current to test the possibility of field reversal due to an ion current layer encircling the axis of a mirror machine. The two-dimensional (r,z) Superlayer simulation code is being used to model the process of current buildup during neutral injection. The code predicts that for idealistic axis encircling orbits an ion layer can easily reverse the field with no problem of particle loss. For more realistic injection scenarios the approach to field reversal is more difficult and in some cases the maximum obtainable current layer strength is below that required for reversal. Even in this event, however, the magnetic well is greatly increased. A sizeable energy spread on the injected beam is usually required to prevent the buildup of large amplitude radial oscillations and large axial loss of particles. Velocity distributions with larger $\langle v_r^2 \rangle$ typically result in lower maximum current strengths. The actual process by which the current strength is limited is not yet well determined due to numerical problems.

*Work performed under the auspices of the U. S. Energy R. & D. Administration.

Prepared for publication in the Proceedings of the Annual Meeting on Theoretical Aspects of CTR, Washington, D. C., April 7-9, 1975.

RECENT RESULTS FROM TOKAMAK MODELING*

Research Staff, Laboratory for Applied Plasma Studies+
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Results are presented from SAI Tokamak modeling codes: G1M is a one-dimensional MHD code which employs a generalized metric, allowing studies of 1D slices through axisymmetric systems of arbitrary cross section. It is a hybrid code including a variety of atomic and other micro-processes in a phenomenological way. GØM is a point model code which examines both equilibrium energy balance and time history of the space average conditions of a Tokamak. Specific results presented will include: i) Modeling of the Texas Turbulent Torus. The modeling includes the effects of ion acoustic and other instabilities which are uniquely relevant to this rapidly pulsed Tokamak. ii) The Effect of Ellipticity on Pfirsch-Schlüter Diffusion. This study produces analytic results for inclusion in non-circular cross section diffusion codes. Applications using G1M will be presented showing the effect of ellipticity on Tokamak startup, as well as on impurity diffusion and accumulation. The latter calculations are relevant to the Doublet IIA experiment. iii) Skin Effect Studies. The G1M code employs a coronal equilibrium model of impurity stripping to calculate the impurity level required to explain the published startup behavior of an ST ("Ontogeny") Tokamak, assuming that no instability processes contribute. The required impurity level is not implausible. iv) Scaling of Tokamak Loss Rates. We use GØM to demonstrate that the space average TFR Tokamak results agree with the predictions of WASH 1295, with trapped electron instabilities producing the dominant electron heat loss. We present point model predictions for other existing experiments, examine the published comparison between the Doublet II and ST Tokamak results, and project GØM into the reactor regime. v) 2D Flux Coordinate Transport Code. Preliminary calculations of Tokamak startup will be presented.

*Research supported by the Energy Research and Development Agency

+R. N. Byrne, A. C. Engebretson, S. Hamasaki, H. H. Klein, N. A. Krall, J. B. McBride, G. W. Stuart, A. L. Sulton, S. Tamor, C. E. Wagner

Finite β Effects on Drift-Dissipative Instabilities

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In a toroidal plasma immersed in a magnetic field of shear length L_s and having a radial density gradient of scale length L_n, finite β (plasma pressure/magnetic pressure) can result in an additional stabilizing influence over and above that due to magnetic shear. These finite β corrections to the finite ion gyration radius and electron temperature gradient dissipative trapped electron and collisionless drift instabilities are evaluated. In the $\beta L_s/L_n << 1$ and $\beta k^2 \rho_i^2 L_s^2/L_n^2 << 1$ limit, where ρ_i is the ion gyration radius and k is the poloidal wave number, the result simplifies considerably and indicates that as $\beta k^2 \rho_i^2 L_s^2/L_n^2 \rightarrow 1$ the $k^2 \rho_i^2$ destabilizing terms are strongly stabilized, while for $\beta << k^2 \rho_i^2 << 1$ the temperature gradient terms are only slightly stabilized. In addition, a significant aspect ratio correction to the collisionless drift stability condition will be discussed, as well as the results of local theory.

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Alfven Wave Turbulence Due To Alpha Particles

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We considered the compressional Alfven wave in an infinite homogeneous magnetized plasma, driven unstable through a wave-particle resonance process by an isotropic but non-Maxwellian distribution of fusion born alpha particles[1]. Keeping $k_\perp \rho_\alpha$ finite (where ρ_α is the alpha particle gyroradius), there exists a spectrum of unstable waves with $\omega < k_\parallel v_\alpha$ (where v_α is the alpha particle speed) possibly producing turbulence scattering of the alpha particles. This is a relevant effect with respect to alpha particle containment and heating in fusion devices. Resonance broadening theory is used to calculate the diffusion tensor elements at saturation in order to determine the dominant saturation mechanism and the ensuing anomalous transport rates.

[1] L.V. Korablev, L.I.Rudakov, Soviet Phys. JETP 27 (1968) 439

Numerical Studies of Continuum Modes in Ideal MHD*

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Our stability code,[1] based upon extremizing the Lagrangian using expansion functions, enables us to generate the complete spectrum of ideal MHD. This provides for a very efficient way to study the properties of the continuous spectra. First, we determine the limits on the continua together with the associated eigenfunctions, the limits being fixed by the radial variation of equilibrium quantities in our model. Next, by introducing ellipticity or a variably placed conducting wall we show how discrete modes interact with the continua; this occurs because the boundary conditions are affected by ellipticity and wall position. Finally, we demonstrate connection between interchange unstable modes and the continua.

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[1] M. S. Chance et al., IAEA Conf. on Plasma Phys. and Cont. Nucl. Fusion Res., Tokyo, Japan, Nov. 1974, paper IAEA-CN-33/A12-4.

* Work supported by U.S.E.R.D.A. Contract AT(11-1)-3073. Use was made of computer facilities supported in part by NSF Grant GP 579.

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Parametric Instabilities and Anomalous Diffusion Near the Upper-Hybrid Frequency*

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Parametric instabilities and associated anomalous particle diffusions due to a pump oscillating near the upper-hybrid frequency are studied in theory and by numerical simulations for a two-dimensional magnetized plasma. It is shown that, besides the three-wave decay process, the convective-cell mode, a purely damped diffusive mode in the absence of pump, can be parametrically driven to be a purely growing mode. Accompanied with the enhanced fluctuations, anomalous particle diffusions are observed. Theory and simulation results are in good agreement.

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Mode Simulation and Theory of Ion Acoustic Turbulence

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A study of turbulent heating is reported using the new dynamical code IONAC which follows the evolution of the spectral distribution of ion acoustic turbulence in k-space. We call the procedure mode simulation. The mode simulation code is based on the renormalized Vlasov turbulence theory[1] in which we sum the principal secularities in the time dependence of the fields and the wave-particle interactions of the Vlasov-Poisson system. Compared to particle simulation the dynamics of unstable systems with physical mass ratios and large $n\lambda_{De}^2$ is inexpensive to follow for long periods ($10^3\ \omega_{pi}^{-1}$). Presently, the code includes the effects of (1) spontaneous emission by the particles into the modes, (2) induced emission and absorption by the particles in the presence of the turbulent fields (effective collisions), (3) induced mode scattering off the dressed particles and (4) suprathermal particle production as calculated by the nonlocal velocity space processes predicted by the renormalized turbulence theory. For the problem of current driven ion-acoustic turbulence an effort is made to compare the macroscopic plasma properties obtained from IONAC with those reported from particle simulation and from turbulent experiments. Two regimes are distinguished and shown on film: (1) that in which the suprathermal ions are confined in which case the heating occurs as a pulse of a $50\omega_{pi}^{-1}$ period and (2) that in which suprathermal ions are not retained (typical of the weakly ionized experiments) in which case a slowly decaying turbulent state with $W/nT_e \sim 10^{-4}$ to 10^{-3} persists over hundreds of ω_{pi}^{-1}.

[1] D. Choi and W. Horton, Jr., Phys. Fluids 17, 2048 (1974).

This work was supported by U. S. Atomic Energy Commission Contract No. AT-(40-1)-4478.

Impurity Transport in Tokamaks for Intermediate-to-High Collision Frequencies

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Neoclassical transport coefficients for ion and impurity species are computed in a large aspect ratio toroidal magnetic confinement system in order to examine the transition between the plateau and Pfirsch-Schlüter regimes. Terms of order $(m_i/m_I)^{1/2}$ are retained and the effect of electrons is neglected. The calculation employs a hybrid identical particle collision operator combining Krook and pitch angle scattering terms with the identical particle collision frequency taken to be a function of velocity which is determined from a fit to known exact results for classical transport. Simple approximate analytic representations of these results are presented.

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MAGNETIC MOMENT NONCONSERVATION IN MIRROR FIELDS*

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Adiabaticity is an important consideration in determining the lifetime of energetic particles (such as those resulting from neutral beam injection) in a magnetic confinement device. Numerical studies[1] indicate that the magnetic moment of a particle may undergo a jump $\Delta\mu$ upon crossing certain planes of symmetry, for example, the midplane of a symmetric mirror field. A procedure for analytically determining $\Delta\mu$ in simple vacuum magnetic field configurations has been developed by Hastie, Hobbs, and Taylor[2] and improved upon by Howard[3] and Krushkal.[4] The theoretical and numerical results indicate that $\Delta\mu$ varies as exp $(-1/\varepsilon)$, where ε is of order of the ratio of the Larmor radius to the field scale length.

In the present work, this procedure is generalized to apply to a much broader class of field configurations, including non-vacuum and minimum-B fields. It is shown that the expression for $\Delta\mu$ reduces to an integral which depends only on particle parameters and the variation of the magnetic field strength along a guiding center path. This is made possible by a general relationship that is shown to exist between the field line curvature and the functional form of the field magnitude in the vicinity of its complex zeros. Under the assumption that the dependence of the field magnitude on position along a field line can be approximated by a function with well-separated zeroes, the integral is evaluated explicitly. A procedure is given for suitably modeling a real field. The theory is applied to simple analytic models of non-vacuum mirror fields and compared with numerical results. The range of validity of the method employed in the present work and in the papers cited above is discussed.

*Work performed under the auspices of the U. S. Energy R. & D. Administration.

[1] A. Garren, et al., Second International Conf. Peaceful Uses Atom. Energy (Proc. Conf. Geneva 31 1958).

[2] R. J. Hastie, G. D. Hobbs, J. B. Taylor, 1969 Plasma Physics and Controlled Nuclear Fusion Research (International Atomic Energy Agency, Vienna) Vol. 1, p. 389.

[3] J. E. Howard, Phys. Fluids, 14, 2378 (1971).

[4] E. M. Krushkal, 1973 Soviet Physics- Technical Physics 17, 1792,

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X-Ray Emission From Laser-Generated Plasmas at 10^{16} W/cm^2

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Experimenters have deposited a few Joules ($>10^{16}$ W/cm^2) of 1.06 μ laser energy on a plane target in 25 psec. They have measured X-ray spectra from 1 to 500 keV and obtained fluxes varying over more than 7 orders of magnitude.[1,2] Computer models have invoked modified electron distributions or anomalously reduced thermal conductivity in an attempt to fit the experimental data.[3,4] We present here numerical results which agree with experiment over the entire range within a factor of 10 and which use only classical transport and maxwellian electron distributions.

The code, which has been described previously,[5] is an Eulerian 2D hydrodynamic plasma laser code to which free-free X-ray spectrum diagnostics and a more complete classical thermal conduction model have been added.[6] Its main features are that it uses an equation of state for the plasma, it uses an energy equation to describe the target energy dynamics and it provides a complete treatment of thermally self-generated magnetic field effects, including the classical thermal conductivity tensor. Thermal flux-limiting has been included without any significant effect. The bremsstrahlung radiation rate has been broken into frequency intervals

$$P_{brem}(E_1,E_2) = P_{b\ell}\, e^{-(E_2-E_1)/k\,T_\ell}$$

where $P_{b\ell}$ and T_ℓ are the local total bremsstrahlung radiation rate and temperature. The radiation output in each frequency range is integrated over space and time to obtain the total output.

The agreement in shape and magnitude between the numerical results and the experimental one is quite good, considering that there are no adjustable parameters. The spectra can be analyzed in terms of the electron temperature and density profiles obtained from the code as a function of time. The hard X-ray component is coming from hot electrons trapped by the large magnetic fields (> 1 MG; $w_{ce}\tau_e \simeq 10^5$) produced near the critical depth. The soft component is coming from a region located just behind the critical depth where the temperature drops well below its maximum value but where the density is much higher. Different physical parameters have been varied without much effect on the X-ray spectra. The only drastic change is obtained when the magnetic field is set to zero in which case the X-ray intensity at 40 keV for example drops 4 orders of magnitude.

From these results, suprathermal electrons seem unnecessary to explain X-ray experiments at high laser irradiances but on the other hand magnetic field effects are very important and dominate transport in laser-generated plasmas especially when conditions exist for which they are very large.

[1] J. F. Kephart et al, Appl. Phys. Lett. 25, 108 (1974).

[2] B. H. Ripin et al, to be published.

[3] H. D. Shay et al, Bull. Am. Phys. Soc. II 19, 868 (1974).

[4] A. W. Ehler et al, Los Alamos Scientific Laboratory Report LA-UR-74-672.

[5] N. K. Winsor and D. A. Tidman, Phys. Rev. Lett. 31, 1044 (1973).

[6] S. I. Braginskii in "Reviews of Plasma Physics," Vol. I, Consultants Bureau, p. 205 (1965).

Resonant Ion Heating and Slide-Away Regime in High Temperature Plasmas*

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We consider a high temperature toroidal plasma subject to a d.c. electric field and in which a significant fraction of the electron population is magnetically trapped. Then the distribution of current-carrying electrons is shown to acquire a positive slope(1) as a function of the longitudinal velocity $v_\parallel$ only if the average electron drift velocity $u_\parallel$ is a finite fraction of the electron thermal velocity v_{the}. Therefore, no current-driven modes, of the types that are commonly considered for the explanation of the observed anomalous electrical resistivity in the absence of magnetic trapping and having phase velocities much smaller than the electron thermal velocity, can be excited for $u_\parallel \ll v_{the}$. When the induced distortion of the electron distribution becomes sufficiently large, corresponding for instance to $u_\parallel/v_{the} \gtrsim 0.1$, modes can be excited in the following frequency ranges: a) the average electron transit frequency or lower, b) the lower hybrid frequency ($\approx$ ion plasma frequency), c) the "reduced" electron plasma frequency ($k_\parallel \omega_{pe}/k$), d) the electron cyclotron frequency. We notice that the excitation of the last type of modes is due to the "loss-cone" feature of the electron distribution developing for velocities opposite to the direction of acceleration and only the first two classes of mode can transfer directly momentum and energy to the ion population.

Experiments carried out on the Alcator device have indicated the existence of a "slide-away" regime, when a finite fraction of the electron population tends to run away as a whole, with remarkable distortions of the electron distribution from a Maxwellian being observed from Thomson scattering and soft X-ray measurements. The onset of strong emission in the last three of the above mention frequency ranges has been detected and a sharp enhancement of the ion temperature has been correlated with fluctuations at about the ion plasma frequency. The relevant analysis indicates that longitudinal energy is absorbed by a resonance of this mode with the circulating electrons and is transferred as transverse (to the magnetic field) energy to the main body of the ion distribution. The measured threshold of excitation of the same mode $-u_\parallel/v_{the} \gtrsim 0.5-$ is quite well reproduced by assuming a distorted equilibrium electron distribution, rather than by a displaced single Maxwellian.

Finally, different theoretical models (Vlasov and Fokker-Planck) adopted for a description of the electron distribution appropriate to the slide-away regime are briefly discussed.

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1) N. Winsor et al., Paper presented at this meeting.

KINETIC AND FLUID-NUMERICAL STUDIES OF HIGH-β PLASMAS*

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The Maryland theory program in high-β plasmas emphasizes kinetic (Vlasov-Maxwell) studies of the equilibrium, stability, and anomalous transport properties relevant to the implosion and immediate post-implosion phases of high-density pinch experiments, and numerical studies of the implosion dynamics using fluid equations modified to include classical transport as well as the anomalous transport produced by various cross-field instabilities. This paper summarizes several recent investigations in this program. These investigations include: (a) Studies of kinetic stability behavior. First, a nonlocal, fully kinetic, matrix dispersion equation has been derived assuming electrostatic perturbations about linear θ-pinch equilibria of the form $f_j^o(H-\omega_j P_\theta, v_z)$. This matrix dispersion equation contains a wealth of new stability information heretofore not accessible analytically. For example, it is shown that a nonlocal version of the lower-hybrid-drift instability persists for low values of azimuthal harmonic number (m=1,2,...) and radial wavelengths comparable with the pinch radius. Second, a new instability is discussed that provides a viable nonlinear mechanism for rapid ion energy isotropization in high-density pinches. Low frequency ($\omega \lesssim \Omega_i$) electromagnetic perturbations propagating parallel to $\underset{\sim}{B}_o$ are shown to exhibit instability for $T_{i\perp} > T_{i\parallel}$. For large anisotropy, and $\beta_{i\perp} \approx 1$, the characteristic maximum growth rate is $\gamma_M \approx \Omega_i$, which is much larger than the growth rate for the mirror instability. (b) Fluid-numerical studies of high-density θ-pinch implosion dynamics. For sufficiently high densities ($n \gtrsim 10^{15}/cm^3$, say) both classical and anomalous transport processes can play an important role in implosion dynamics. The transport code of Liewer and Krall (which includes anomalous transport processes in a self-consistent manner) has been modified and generalized to include the effects of classical transport (frictional force, thermal force, heat conduction, heat convection, resistive heating, and thermalization) as well as the effects of neutral particle diffusion, ionization and charge exchange. The numerical results are found to compare quite favorably with the observed implosion dynamics in high-density pinch experiments, such as Scylla 1-B and Scyllac. The specific processes that are important during the implosion depend on initial ionization level, bias field, radial location, etc. (c) The development and application of a hybrid-kinetic model (Vlasov ions and drift-kinetic electrons) to describe the equilibrium and stability properties of collisionless high-β plasmas. Since the ion orbits may be large in high-density pinch experiments, the ions are described by the exact (unexpanded) Vlasov equation. The electrons are assumed to be strongly magnetized with $\omega/\Omega_e \sim r_{Le}/L_1 = \varepsilon << 1$, and the electron Vlasov equation is expanded to give a drift-kinetic equation for f_e. Complex equilibrium configurations (toroidal geometry, arbitrary cross section, etc.) can be described within the context of the hybrid-kinetic model with considerable flexibility, a feature which is not true in a fully kinetic description. Stability calculations for linear theta-pinch geometry show that the hybrid-kinetic model correct to $O(\varepsilon)$ has sufficient physics content (parallel kinetic effects, electron polarization drift, etc.) to describe implosion stability behavior on the fast time scale $\omega_{LH}^{-1} = (\Omega_e\Omega_i)^{-1/2}$.

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ANOMALOUS TRANSPORT PROPERTIES CHARACTERISTIC OF THE POST-IMPLOSION PHASE OF HIGH-DENSITY PINCH EXPERIMENTS*

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The immediate post-implosion phase of the ZT-1 toroidal screw pinch experimen is characterized by a sufficiently high anomalous resistivity and turbulence level that the average density and magnetic field profiles eventually distort from an MHD stable configuration to a regime where plasma confinement is lost (probably through m=1 kink instability). Moreover, linear theta-pinch experiments during the late stages of implosion often exhibit considerably more diffuse magnetic field profiles than predicted by existing theoretical estimates and fluid-numerical studies that anomalous transport processes only for $V_E > v_i$ (here v_i is the ion thermal velocity V_E is the cross-field electron $\underset{\sim}{E} \times \underset{\sim}{B}$ drift velocity). These results indicate that substantial anomalous transport persists in the low-drift-velocity regime $V_E \lesssim v_i$, generally characteristic of the late-implosion and immediate post-implosion phases of high-density pinch experiments. This paper summarizes the results of extensive analytic and numerical studies of (a) microinstabilities driven by cross-field electron current, and (b) the associated anomalous transport properties. Primary (although not exclusive) emphasis is placed on the low-drift-velocity regime ($V_E \lesssim$ which has heretofore not been adequately investigated in the literature. For stro magnetized electrons and unmagnetized ions it is found that maximum growth occurs for flute-like perturbations ($\underset{\sim}{k}\cdot\underset{\sim}{B}_o=0$), and the dominant instability is identified as the hot-ion lower-hybrid-drift instability (driven by V_E, $\partial n/\partial x$ and $\partial B/\partial x$). Nonlinear estimates of the instantaneous heating rates and rate of momentum transfer are made, and the results are studied numerically to determine the parametric dependence on V_E/v_i and the level of turbulent field fluctuation energy ε_F. Fowler's thermodynamic method is used to obtain a nonlinear bound on ε_F, and hence on transport properties such as the anomalous resistivity η_{an}. It is shown that the lower-hybrid-drift instability can result in substantial resistivity and plasma heating for $V_E \lesssim v_i$, as well as for the large-drift-velocity regime ($V_E > v_i$). For exampl when $T_i/T_e >> 1$ and $\omega_{pe}^2/\omega_{ce}^2 >> 1$, the bound on anomalous resistivity for $V_E \lesssim v_i$ i

$$[\eta_{an}]_{MAX} \simeq 4\pi\sqrt{\frac{\pi}{2}}\left(\frac{V_E}{v_i}\right)^2 \frac{\omega_{LH}}{\omega_{ce}^2}$$

where $\omega_{LH} = (\omega_{ci}\omega_{ce})^{1/2}$ is the lower-hybrid frequency. This large value of resistivity is consistent (in a local sense) with the value of resistivity inferre from experimental measurements made during the post-implosion phase of ZT-1.

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Some New Ideas on Wet Wood Burners

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Recent experiments[1] have shown that in some Tokamak experiments the electron heat transport can be many times faster than is predicted by neo-classical or pseudoclassical theory while the ion transport remained neo-classical. Some recent computer simulations[2] have shown that collective effects can greatly enhance the electron heat transport while the ion transport remains unchanged. In addition the electrons can suffer heavy energy loss due to impurities[3]; as little as 0.1% high z impurity significantly alters the energy balance and sufficient impurity levels will prevent ignition. Under these conditions the TCT concept is appealing because it can reach break-even with plasma conditions not much different than have been achieved in existing Tokamaks. However, the maximum energy multiplication one can achieve is 3 ∿ 4. A modification of this concept is the clamped hot ion concept where energy multiplications of 6 ∿ 8 can be achieved. One would like to have energy multiplications of 10 or greater. We have investigated the gains one might achieve if one fed energy directly to ions so as to maintain a high ion temperature. The ions are assumed to lose energy to colder electrons which have a high rate of energy loss due to anomalous processes or to impurity radiation. We assume that the ion heat conduction losses can be kept below the ion electron energy exchange time. We find that if these conditions can be obtained large yield ratios can be achieved. For one calculation holding the ion temperature at 20 kev with an electron temperature of 10 kev gave energy multiplications of 10. The required energy confinement time for the electrons was $n\tau = 2 \times 10^{13}$. The fact that one can achieve good yields with heavy electron energy loss means that one can consider many new types of reactors some of which will be discussed.

Work supported by ERDA AT(04-3)-34 P.A. 157.

[1]Papers A6-1 and A6-2, The 5th International Conference on Plasma Physics and Controlled Nuclear Fusion Research Tokyo, 11-15 November 1974; K. Bol et al., Phys. Rev. Lett. 32, 661 (1974).

[2]C. Chu, J. M. Dawson and H. Okuda, PPG-208, Submitted to Phys. of Fluids.

[3]R. Conn and J. Kesner, Submitted to Nuclear Fusion.

Secular Mode Coupling and Drag
in the Electron Beam-Plasma Instability

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A new theoretical description of the breakup of large scale oscillations of particles in turbulent plams into spatially local oscillations (clumps) is presented and the importance of this process for the evolution of the beam-plasma instability is investigated numerically. The spatially local oscillations or "clumps" emit waves in a manner which is analogous to the discrete particle wave emission in thermal equilibrium and can be represented by a nonlinear source. The particles also experience an anomalous drag as they emit waves, which is analagous to the discrete particle drag. The reduction of an individual wave to a spatially localized oscillation and concurrent emission of new waves represents a mode coupling process which couples all modes in the plasma. The mode coupling and drag are included in a numerical calculation of the evolution of the electron beam-plasma instability for a weak thermally spread beam (bump-on-tail distribution). The wave energy spreads toward high phase velocity modes while the particle velocity distribution evolves toward a state characterized by the bunching of particles in velocity space in the resonant region of the largest amplitude waves.

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Modifications to Quasilinear Theory in Turbulent Plasma with Numerical Application to the Electron Beam-Plasma Instability

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We include the stochastic orbits of particles in phase space in investigating the energy exchange between waves and particles in turbulent plasma. Our final equations resemble those of quasi-linear theory with the important difference that, in the expressions for $\varepsilon(k,\omega)$ and $\partial f/\partial t$, $\partial f/\partial v$ must be replaced by $\langle \partial f/\partial v \rangle_V$, where $\langle\ \rangle_V$ represents a velocity space average over a region of width v_D, the velocity uncertainty of a particle after a diffusion time. The equation for the time evolution of f(v,t) is consequently a non-local functional of $\partial f(\bar{v},t)/\partial \bar{v}$ and does not simply reduce to a Fokker-Planck equation as predicted by other authors[1,2]. Unlike other theories[1,2], our final equations conserve energy exactly. The results of a numerical calculation using these equations for the one-dimensional electron beam-plasma instability with fixed ions are shown. The total wave energy strongly overshoots its steady state level, a phenomenon frequently observed in computer simulations. In the final state the particle velocity distribution forms an irregular plateau in the resonant region of velocity space.

[1]Dupree, T.H., Phys. Fluids 9, 1773 (1966).
[2]Weinstock, Jerome, Phys. Fluids 12, 1045 (1969).
Work supported by ONR, ERDA and NSF.

The Destruction of Magnetic Surfaces in Tokamaks*

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We report investigations of the feasibility of the hypothesis that the disruptive instability in tokamaks is caused by the breakup of magnetic surfaces. The envisioned breakup is caused by the interaction of m = 2, n = 1 and m = 3, n = 1 helical perturbations or by the interaction of the m = 2, n = 1 perturbation and the m = 1, n = 0 toroidal field (with R/a = 5). Both numerical and analytic work has been done for two MHD equilibrium current profiles: parabolic $[q \sim (1 - r^2/2a^2)^{-1}]$ and the peaked model $(q \sim 1 + \lambda r^2/a^2)$. For the parabolic profiles, in agreement with previous work[1] perturbation strengths of 1.5% are required to break up surfaces when m = 2 and m = 3 perturbations are present. When the m = 2 perturbation alone interacts with the toroidal field, perturbation strengths greater than 4% are required. For the peaked model with q(0) = 1.1, q(a) = 3.7, perturbation strengths of 0.5% were found to be sufficient to break the surfaces when both m = 2 and m = 3 perturbations are present, whereas strengths of 1.5% are required when only the m = 2 perturbation interacts with the toroidal field. These perturbation amplitudes are in reasonable agreement with the experimental measurements of von Goeler[2] on the ST tokamak. The peaked model permits breakup of surfaces for smaller perturbation strengths primarily because of its greater shear.

[1]P. Chrisman, J. Clarke, and J. Rome, ORNL-TM-4501 (March 1974).
[2]S. von Goeler, Bull. Am. Phys. Soc. II, 19, 883 (1974).

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ELECTRON BEAM FUSION

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ABSTRACT

Two programs[1,2] are presently studying the feasibility of pulsed fusion initiated by relativistic electron beam implosion of pellets. The success of these efforts will be determined by the ability to focus the 10^{13-15} Watt beams to millimeter focal diameters, and to achieve efficient, symmetric energy deposition in the spherical D-T gas filled targets. Theoretical studies of the focusing process in intense electron beam diodes using a 2-D particle simulation model have been reported.[3] Recent extensions of this approach will be described, including the effects of anode-emitted ion flow on the diode pinching behavior. Emission laws for the electrons accelerated from the cathode plasma have been reformulated to include the effects of ion flow and the transverse magnetic field. Initial comparisons with temporally- and spatially-resolved current density data from a Hydra (600 kV, 250 kA, 100 ns) beam pinching experiment will be discussed. These results show that ion effects must be included to obtain reasonable correlation.

The symmetry of energy deposition for super-pinched-beam irradiated spherical targets has been studied using a relativistic fluid formalism. Earlier theoretical work[4] proposed a hot electron gas model based on fluid envelope calculations and compared the focusing behavior with particle simulations. Recent work will be described which places constraints on the beam drift velocity and temperature needed to achieve a specified degree of isotropy.

A surprising degree of symmetry has been observed in flash x-ray photographs and time-integrated x-ray pinhole pictures from recent implosion experiments on Hydra. The experiments employed hollow gold spheres slightly elevated from the anode. This tendency towards symmetric deposition, consistent with a fluid-like beam behavior, was also implied by the much reduced anode damage when compared with earlier hemisphere and flat-anode cratering studies. Two-dimensional hydrodynamic code calculations assuming classical collisional deposition have been employed to study this phenomenon. Cases were run for a variety of assumptions on energy deposition symmetry including preferential deposition on the cathode pole of the sphere and in the equatorial regions of the sphere, consistent with a primarily radial electron flow. Results will be presented[5] which indicate that correlation with the anode damage and the flash x-ray photographs can be obtained only for symmetric or near-symmetric deposition.

[1] J. Chang, et al., "Relativistic Electron Beam Induced Fusion,"Fifth Conference on Plasma Physics and Cont. Nuc. Fusion Research, Nov. 11-15, 1974, Tokyo, Japan.

[2] Yu. V. Koba, et al., "Study of the Focusing of High-Current Beams for Heating and Compression of a Dense D-T Plasma," Fifth Conf. on Plasma Physics and Cont. Nuc. Fusion Research, Nov. 11-15, 1974, Tokyo, Japan.

[3] G. Yonas, J. W. Poukey, K. R. Prestwich, J. R. Freeman, A. J. Toepfer, M. J. Clauser, Nuc. Fusion 14, 731 (1974); J. W. Poukey, Bull. Am. Phys. Soc. 19, 870 (1974).

[4] J. Poukey and A. Toepfer, Phys. Fluids 17, 1582 (1974).

[5] J. Chang, M. M. Widner, G. W. Kuswa, G. Yonas, submitted for publication.

Unstable Electronic Loss Cone Modes

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Turbulence associated with unstable electron waves may give rise to the anomalous cross-field transport of electrons which has been suggested as a possible explanation for the non-occurrence of drift cyclotron loss cone mode in the 2X and PR experiments. These waves could also contribute to the anomalous electronic energy transport observed in Tokamaks. Since direct observation of these waves and the associated transport may be difficult in large confinement geometries, we have made a detailed calculation of growth rates, using the linear dispersion relation, to determine whether observation of the growth, saturation and anomalous transport, if any, would be feasible using available laboratory plasmas, such as the SCAMP (Source Chamber and Magnetized Plasmas) at UCLA. It appears that for typical laboratory conditions ($B = 1$ kg; plasma dimensions $\gtrsim 10$ cm. diameter × 1 m length), the wave growth should be observable (with e-folding lengths of a few centimeters). For fixed wavelength parallel to $\underline{B}$ (determined by a suitable antenna), and given values of mirror ratio, R; ambipolar potential, ϕ; and density, the growth rate is a sharply peaked function of excitation frequency, ω; the optimum value of $\nu = \omega/\omega_{ce}$ varies from .65 to .48 as R varies from 1 to 5. With $k_z r_{ce}$ of order 0.25, spatial growth rates $\mathrm{Im} k_\perp / k_\perp$ or order 10^{-2} are possible with $\psi \gtrsim 3T_3$. The calculations are greatly simplified by the fact that for $0.5 \lesssim \nu < 1$, the $n = 1$ term in the Bessel function series for the dielectric function (corresponding to the fundamental cyclotron frequency) dominates the expression for $\mathrm{Im}\varepsilon$.

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EQUILIBRIUM AND STABILITY OF HIGH-BETA DIFFUSE TOKAMAKS*

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The limitations on β as regards equilibrium and stability of tokamaks are investigated. The regime of interest is obtained for $\beta \sim \varepsilon$ and $B_{pol}/B_{tor} \sim \varepsilon$, where ε is the inverse aspect ratio. The shift δ of the magnetic axis away from the center of the tube is then of order unity so that the equilibrium is essentially two-dimensional. (In the usual low-β tokamak ordering where $\beta \sim \varepsilon^2$, the shift $\delta \sim \varepsilon$ so that one-dimensional approximations are possible.) After Fourier decomposing modes in the ignorable angle ϕ the stability analysis also becomes two-dimensional, so that the basic problem consists of calculating equilibria accurately enough that it makes sense to do a spectral analysis of the stability.

The nonlinear equilibrium equation for the flux function ψ is solved for exponential pressure profiles, exploiting a conformal mapping to a polar coordinate system r', θ' in which the magnetic axis is at the center $r' = 0$ and the plasma boundary at the circle $r' = 1$. Both equilibrium and stability are then solved in this coordinate system so that no accuracy is lost in converting from one coordinate system to another. This transformation makes the low-β equilibrium trivially concentric circles to leading order, whereas high-β equilibria can be accurately represented by an extremely rapidly converging Fourier-series in the angle θ'.

Limits on the equilibrium typically arise from the appearance of a second magnetic axis and bifurcated solutions when δ becomes too large (this occurs at about $\delta \approx 0.4$). This limit fixes a critical value of β depending on the safety factor q. From the stability analysis similar limitations on β arise so that one can then optimize parameters so as to obtain the maximum possible β as far as ideal MHD equilibria and stability are concerned.

$\beta \simeq 1.8\%$ (?)

* Work supported by the AEC, Euratom, FOM, and ZWO.

**On leave of absence from FOM, Jutphaas, The Netherlands.

Instability of Energetic Ion Beam Injection

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Currently there is considerable interest in the use of energetic ion beams to heat magnetically confined plasmas to ignition temperature. An uncertainty in this heating technique is the possible role of instabilities driven by the fast ion beams. For this reason, we have investigated homogeneous plasma modes, including the lower-hybrid wave and the beam ion cyclotron wave, as well as inhomogeneous plasma modes, including the trapped electron drift wave, which taps the free energy associated with the beam anisotropy. The growth rates of these modes are calculated and the stability thresholds are compared to parameters for both present injection experiments and the proposed TFTR device.

* Center for Theoretical Physics Fellow

Quasi-Static 2D Model for the Evolution of a General Tokamak Cross-Section*

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A computer code currently under development at NRL computes the time history of a general 2D Tokamak cross-section in a quasi-static approximation which removes MHD modes of oscillation through neglect of the inertial terms. Thus the timesteps for the calculation are limited only by diffusive and plasma transport effects and the complete evolution of an entire discharge can be followed accurately and efficiently. A Lagrangian triangular mesh[1,2] is adopted where the motion of the grid vertices is chosen to ensure a local quasi-static force balance with $\underline{J} \times \underline{B} = \underline{\nabla} P$. The triangular grid is sufficiently flexible to permit complicated magnetic field-plasma-vacuum-wall configurations with separatrices, axes, and magnetic surfaces. Magnetic islands and multiply-connected regions can also be treated.

Many of the algorithms being incorporated into the code are being tested in a similar quasi-static 1D code. Details of the triangular grid scheme will be presented with emphasis on the possible generalizations of physics and geometry to problems of interest to the CTR community. The results of several time dependent problems will be presented in which plasma calumns are confined by a conducting wall.

*Work supported in part by ERDA

1. J. P. Boris, A. E. Robson, R. A. Shanny, P. J. Turchi, N. K. Winsor, BAPS, paper 3G1, pg. 1299, October 1973.
2. J. P. Boris and K. L. Hain, BAPS, paper 6G1, pg. 942, October 1974.

Resistive Instabilities in a Tokamak*

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The stability of a tokamak against resistive fluid modes is determined as an application of a general theory of resistive instabilities in a torus.[1] Analytic expressions for the critical parameters D_R, H, and Δ_c of Ref. 1 are obtained by means of a large-aspect-ratio expansion in which the lowest-order magnetic surfaces are concentric circular cylinders and the current and pressure profiles are arbitrary. Numerical evaluation is made when the pressure p and safety factor q vary quadratically with radius. To complete the analysis we need Δ, the jump in the logarithmic derivative of the normal perturbed magnetic field, used to match to the ideal region outside the singular layer. The values obtained for a zero-pressure, cylindrical tokamak model with the same current profile[2] are correct to lowest order in the aspect ratio expansion. Evaluation of the stability is made for the $m = 2$ and 3 modified tearing modes, which are generally associated with Mirnov oscillations[3] observed in smaller machines like T-3, ST, and ORMAK. We find these modes to be unstable in such devices, but stabilized by curvature effects in smaller-aspect-ratio, hotter devices like PLT. Introduction of the Hall term into Ohm's law is being studied. It stabilizes one of the two unstable modes, destabilizes the other, and converts its real frequency to the electron diamagnetic frequency, as observed.

[1] A. H. Glasser, J. M. Greene, and J. L. Johnson, MATT-1068 (Aug. 1974).

[2] H. P. Furth, P. H. Rutherford, and H. Selberg, Phys. Fluids 16, 1054 (1973).

[3] S. V. Mirnov and I. V. Semenov, in Plasma Physics and Controlled Nuclear Fusion Research (IAEA, Vienna, 1971) Vol. II, p. 401; Zh. Eksp. Fiz. 60, 2105 (1971), [Sov. Phys. JETP 33, 1134 (1971)].

* Work supported by U.S.E.R.D.A. Contract AT(11-1)-3073.

ADIABATIC AND RESISTIVE ISLAND BEHAVIOR OF DOUBLET AND TOKAMAK

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The behavior of a plasma equilibrium subject to adiabatic constraints[1,2] (also resistive diffusion without inertia[3,4,5]) is governed by an unusual new type of Generalized Differential Equation for which practical analytic and numerical methods are being developed; some Belt Pinch and Doublet results have been previously described[2]. A number of new results are presented here. For example, the current density in a boundary layer near the separatrix is found to vary as $V^{-1/3}$ (V is the volume of a flux surface, measured from the separatrix). Among the implications is that the safety factor, q, remains finite at the separatrix. This has stability implications.

In squeezing a Belt Pinch into a Doublet, several paths of bifurcation and islation have been observed. In one sequence, the first occurrence is a bifurcation, leading to a (two-dimensional) stable asymmetric but topologically simple configuration; this later islates (uniquely) to an asymmetric Doublet. By enforcing symmetry a symmetric islation is observed at a larger value of the compression parameter than the asymmetric islation.

Preliminary results, analytic and numerical, will be described for nonlinear tearing (Mirnov islation), internal kinks (islated bifurcation), and limiter induced islands in cylindrical and helical versions of a Tokamak. A survey will be given of the analytical and physical connections between the simple adiabatic problem, adiabatic islation (in which resistivity is implicit but not overtly used), and the fully resistive treatment without inertia. A complete set of generalized adiabatic jump conditions will be described; the results (also the conservation laws from which they are derived) differ from 2D to axial symmetry, $\gamma = 2$ to $\gamma \neq 2$, symmetry vs. asymmetry, shrinking vs. growing islands, global adiabatic transitions vs. continuous flows.

1. H. Grad, CTR Theory, Berkeley, April 3-5, 1974.
2. H. Grad, P.N. Hu, D.C. Stevens, D.P.P.; Albuquerque, Oct. 28-31, 1974.
3. H. Grad, J. Hogan, Phys. Rev. Letters, June 1970.
4. H. Grad, ORNL Memo, July 1970.
5. H. Grad, Math. Congress, Nice, Sept. 1970.

ABSTRACT

STABILITY OF THE STRONGLY NON-CIRCULAR HIGH-β TOKAMAK

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The stability of a strongly non-circular high-β Tokamak is studied by an asymptotic expansion in three parameters, the toroidal curvature κ, plasma β, ratio of maximum and minimum minor radii of the plasma. For any ordering there are two distinct classes of equilibria with relatively higher and relatively lower current. The higher current equilibria are characterized by weak toroidal effects while the lower current equilibria have strong toroidal effects. The stability analysis is done based on the same asymptotic expansions. Two major classes of potentially unstable modes appear, one in which the instability scales with the equilibrium and one in which the spatial variation is more rapid than in the equilibrium. The latter class of modes is shown to be either stable or readily stabilized. This work principally concerns the former class of modes. It is first shown analytically that the relatively low current equilibria with significant toroidal effects on equilibria and stability are strongly unstable. In fact, in the ordering chosen, the growth rate is formerly unbounded. The numerical work treats a few high current profiles and gives stability conditions, growth rates and eigenfunctions. A vacuum region is included outside the plasma. The procedures used make it relatively simple to do parameter studies and explorations of profiles and a limited amount of such information is presented.

WHAT'S MISSING IN DRIFT-CYCLOTRON LOSS-CONE INSTABILITY*

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The clear disagreement between theoretical prediction and experimental non-observance of the drift-cyclotron loss-cone (DCLC) instability has received much attention over the years and is reported annually at this meeting. In fact, however, one ubiquitous feature of previous calculations is the use of the so-called "asymptotic fluid" approximation for the electron dielectric response function.[1] We conjecture that a reconsideration of the electron dynamics may resolve the dilemma. Two features are omitted from most DCLC analysis, in which the usual treatment[2] first writes Poisson's equation in the form

$$d^2\tilde{\Phi}/dz^2 = G_e + k_\perp^2 D\tilde{\Phi} \ , \quad D = 1 + D_i + (\kappa k_y/k_\perp^2)\omega_{pe}^2/\omega\Omega_e$$

where G_e is the orbit integral along magnetic field lines of the electron motion, D_i is the ion dielectric response, and the last term in D is the electron drift response perpendicular to $\underset{\sim}{B}$. The omissions occur when G_e is replaced by $(d/dz)[(\omega_{pe}^2/\omega^2)d\tilde{\Phi}/dz]$, and neglect (i) the effects of electrons near their turning points [whose reactive response[3] (capacitive) is opposite to that of the speedy (inductive) electrons] and (ii) Landau damping, which is non-local and dissipative. The omissions are most serious in regions of strong gradients of the macroscopic confining potential. Our preliminary analysis indicates that Landau damping tends to stabilize large $k_\perp$ modes, since at large $k_\perp$, $Im(D)/Re(D) \rightarrow 0$, and $Im(D_i)<0$ is the drive which must overcome Landau damping. Modes with $k_\perp a_i$ of order unity may exist if the plasma is well insulated from end walls, but a finite-temperature finite-density external plasma provides stabilization, in agreement with experimentally observed tendencies.[4] Our examination is consistent with the model of 2XII behavior that interprets the experimental operation in terms of a tradeoff between charge exchange decay (quiescent decay) and the need for external plasma to stabilize DCLC modes.[5]

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*Work Performed under the auspices of the U.S. Energy R. & D. Administration.

[1] H. L. Berk and L. D. Pearlstein, Phys. Fluids 14, 1810 (1971).

[2] D. E. Baldwin, et al., Madison Conference (IAEA, Vienna, 1971) V. 2, p. 735.

[3] L. S. Hall and W. Heckrotte, Phys. Fluids 9 1496 (1966).

[4] T. C. Simonen, Lawrence Livermore Lab. Rpt. UCRL-76115 (1964).

[5] L. S. Hall, to be published.

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NUMERICAL MODELING OF THE GAS EMBEDDED Z-PINCH*

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One and 3-D Lagrangian and 2-D Eulerian MHD codes are used to study the gas-embedded Z-pinch. In 1-D, dissociation, ionization, thermal and electrical conductivity, and α-particle heating effects are included. Reasonable agreement with experiment is obtained for snow-plow pinches and exploding wires and tubes. For some cases of slower pinches, (adiabatic in the sound transit time across the layer) while general agreement is obtained, a skin current is calculated in apparent variance with experiment. Calculations of DT reactor-regime pinches where α-particle heating is important will be presented.

In 2-D calculations the skin current distribution is unstable and relaxation by the sausage instability is calculated. End effects are modeled in 2-D (including the possibility of the Nernst-effect). It is found that the hot plasma is lost at a velocity of 0.1-1 of v_{sound} depending on the end boundary conditions.

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Electrostatic Trapping Effects on Neoclassical Transport in Impure Tokamak Plasmas

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It has been noted[1] that the impurity concentrations typical of present tokamak experiments can lead to substantial poloidal variation of the electrostatic potential. Such poloidal electric fields strongly influence the radial transport of particles and energy, and the plasma electric current, both directly, by affecting the guiding center drifts, and indirectly, by causing a poloidal variation of the effective Z for collisions, Z_{eff}. The present study of these effects differs from one presented previously[1] mainly because the impurities are now, more consistently, treated as being in the Pfirsch-Schlüter collision frequency regime; both hydrogen ions and electrons are assumed to be in the plateau regime. For simplicity, the case of only a single impurity species, in sufficient concentration to give $Z_{eff} \sim 3$-5, is considered. It is found that $Ze\tilde{\Phi}/T_i$ can be of order unity, where (Ze) is the impurity charge and $\tilde{\Phi}$ is the poloidally varying part of the potential, as calculated from charge neutrality and force balance considerations. A complete set of transport coefficients is then derived from the drift kinetic equation, for large aspect ratio and several values of Z_{eff}. The various $\tilde{\Phi}$ contributions to these coefficients are not small, and are expected to alter significantly the predictions of neoclassical transport theory in an impure plasma.

[1] R.D. Hazeltine, A. A. Ware, et al., Proc. 5th Int. Conf., Tokyo, 1974.

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SINGLE PARTICLE DRIFT ORBITS IN A FINITE BETA SCALAR PRESSURE EQUILIBRIUM FOR EBT*

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ABSTRACT

Single particle orbits play a central role in the stability and transport analyses of a plasma. In the ELMO Bumpy Torus, finite beta changes the magnetic field in ways which have a profound effect upon single particle drift orbits. To demonstrate some of these effects we have considered a relatively simple equilibrium: scalar pressure peaked on axis ($\beta \simeq 50\%$).

In general the self-dug well created by finite beta reverses the grad-B drifts which dominate the drift motion for trapped particles as well as for some passing particles. For particles with $v_\perp \simeq 0$ (i.e., $v_\parallel \simeq v$) the drift motion is dominated by curvature which is relatively insensitive to finite beta. However, these particles can be affected by ambipolar electric fields and we shall show the effects of an ad-hoc electric field.

We find it convenient to determine, for each pitch angle, the area of the last drift contour which does not intersect the wall of the device. The minimum in this curve is changed by finite beta so that it occurs in the passing region of velocity space rather than in the transition zone as was the case for the vacuum field. While the minimum value is also increased by finite beta, we expect this effect to be more pronounced for the more realistic tensor pressure equilibrium calculations that we are presently pursuing.

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MHD Instabilities of Diffuse Helical $\ell = 1$ Equilibria as an Initial Boundary-Value Problem+)

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The aim of this work is to numerically investigate the gross MHD instabilities of high-ß helical $\ell = 1$ equilibria; this problem will be tackled experimentally in the HBS device /1/.

The method used for the instability calculation is essentially the same as in /2/: The ideal MHD equations are linearized about a given equilibrium and are Fourier-analyzed along the ignorable direction of the equilibrium. The boundary of the domain in which the calculation is performed coincides with the outermost flux surface of the equilibrium. The boundary conditions are such, that the instabilities can be classified as fixed-boundary modes. The initial conditions are arbitrary distributions of the perturbed velocity field, which resemble roughly the motion of a particular azimuthal m-mode or a mixture of m-modes. This two-dimensional initial boundary-value problem is solved numerically by an explicit difference method. The general behaviour of the solution in time shows that stable modes oscillate whereas unstable modes grow exponentially. To exhibit the eigenfunctions of the stable modes and to improve the calculation for unstable modes, a spectral shift technique /3/, /4/ is used. After some time steps the most unstable mode overwhelms all other motion; this dominant mode is described in terms of the corresponding eigenfunctions $\vec{v}^1$, p^1, $\vec{B}^1$, and the growth rate γ of this mode can be determined.

This method is used for studying the stability behaviour of diffuse $\ell = 1$ helically symmetric equilibria with the longitudinal net-current vanishing on every flux surface /5/. The coordinate system in which the stability problem is solved is an (x,y) - coordinate system rotating around the z-axis with a periodicity number h, the reciprocal of the helical wavelength of the equilibrium. No expansions in geometric parameters or physical parameters (ß) are necessary. The calculation is in principle not restricted to a special cross-section.

For helical equilibria, characterized by the parameters $\beta \approx 0.5$, $\delta \approx 1$, $h \approx 0.1$, the $m = 1$ and $m = 2$ modes have been calculated and profiles of the perturbed pressure as well as field-patterns of the velocity and the perturbed magnetic field are shown. The growth rates of these modes are calculated and the dependence on the variation of the parameters is investigated.

+) This work was performed under ther terms of the agreement on association between the Max-Planck-Institut für Plasmaphysik, Garching, and EURATOM.

References

/1/ E. Fünfer, J. Gruber, M. Kaufmann, W. Köppendörfer, J. Neuhauser, "Wall-Stabilized High-Beta Stellarator", IPP 1/139 (1974)
/2/ G. Bateman, W. Schneider, W. Grossmann, Nucl. Fus. 14, 669 (1974)
/3/ A. Sykes, J.A. Wesson, Nucl. Fus. 14, 645 (1974)
/4/ K.U. von Hagenow, private communication
/5/ B. Marder, "$\ell = 1$ Helical Equilibrium", private communication

VLASOV CONFINEMENT FOR HIGH-β PLASMA

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A new method for generating realistic Vlasov confinement equilibria has been devised. Instead of assuming a distribution function which is a given function of the constants of the motion (and which leads to nonlinear equations for the electric and magnetic potentials) we choose density and temperature profiles, motivated by experimental results, and deduce simple formulas for the fields and linear equations for the distribution functions. The steps of this process (expressed here in Cartesian x-y geometry, but analogous steps prevail in other geometries) are as follows. (1) Assume ion density and ion and electron temperature profiles as a function of x to correspond to the desired physical situation. (2) Assume the ions carry no current. The ion distribution function is then a function only of the energy; i.e., the ions are electrostatically confined. The confining field may be obtained immediately as: $E_x(x) = [1/n_i(x)]d[n_i(x)T_{ix}(x)]/dx$. (3) Knowing this electrostatic field and the ion density, the electron density is obtained from Poisson's equation: $n_e(x) = n_i(x) - dE_x(x)/dx$. (4) The magnetic field required to contain $n_e(x)$ in the presence of the electric field is obtained by combining the pressure balance equation with Ampere's law:

$$B_z(x) = B_z(o) - \frac{1}{c}\left(\{2[n_e(\xi)T_{ex}(\xi) + n_i(\xi)T_{ix}(\xi)] - E_x^2(\xi)\}_o^x\right)^{1/2}$$

(5) The ion and electron distribution functions are now found by either an integral equation method or by a series expansion procedure. The former assumes a general dependence of the distribution function on the canonical y momentum and one obtains, in the case of constant temperature profiles, the Fredholm equation

$$n_e(x) = e^{-\phi}\int_{-\infty}^{\infty} dV_y\, e^{-V_y^2/2}\, \psi(V_y - A_y)$$

where ψ is the unknown part of the distribution function. The above equation can be shown to have a unique solution; in addition, a physically acceptable equilibrium solution requires ψ to be everywhere non-negative. In the series method one expands the distribution functions in Hermite functions of the velocit components and obtains recurrence relations from the Vlasov equation for the expansion coefficients. Again nonnegativity is required for equilibrium. One consequence of solutions obtained by this second method is that in general, $T_y(x)$ differs from $T_x(x)$ for both ions and electrons implying the possibility of temperature anisotropy between the azimuthal and radial directions in high-β pinch equilibria. Several examples will be presented.

Effects of Finite Collisionality on

the Trapped Electron Instability in Tokamaks

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The local dispersion relation for electrostatic drift waves has been analyzed in the hydrodynamic ion, adiabatic electron approximation. The nonadiabatic part of the electron density perturbation is an integral operator M on the wave potential, whose matrix elements M_{mn} determine the local growth rate from perturbation theory. These matrix elements have been calculated by solving the electron drift kinetic equation with the Lorentz pitch-angle scattering collision term. Analytical solutions have been obtained in two limiting cases: (1) $\nu_{eff} << \omega_{be} |m_c - \ell q|$, (where m_c = integer closest to ℓq) appropriate for high-energy electrons, and (2) $\omega_{be} |m_c - \ell q| << \nu_{eff}$ (but $\nu_{ei} << v_{th} |m_c - \ell q|/qR$), for low-energy electrons. Numerical solutions are being pursued. An interpolation formula, joining the analytical solutions, is used for the intermediate case, and the matrix elements are then computed by integrating over all electron energies. Using an approximate fit to the result, the growth rate is

$$\gamma = \frac{qR}{v_{the}} \omega_*^2 \eta \left[\frac{a}{\nu_{*e} + b \nu_{*e}^{\frac{3}{2}}} - \frac{c}{|m_c - \ell q|} \right]$$

where ν_{*e} is the electron collisionality parameter, a = 2.9, b = 0.06, $c = \pi^{\frac{1}{2}}/2$. In the limit $\nu_{*e} << |m_c - \ell q|$, the trapped electron mode is recovered, with a boundary-layer correction (coefficient b). In the opposite limit, the collisionless drift mode is obtained. The local growth rate of the trapped electron instability ($\eta \equiv \frac{\partial \ell n\, T_e}{\partial \ell n\, n_e} > 0$) becomes negative in regions sufficiently close to a rational surface that $|m_c - \ell q| < 0.3\, \nu_{*e} + 0.02\, \nu_{*e}^{\frac{3}{2}}$, because of electron Landau damping. A nonlocal analysis is needed in order to determine the net stabilizing effect.

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A MECHANISM FOR NEGATIVE ION FORMATION IN THE SURFACE-PLASMA NEGATIVE HYDROGEN ION SOURCE*

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Within the last few years a magnetron type negative hydrogen ion source has been developed by Belchenko, Dimov, and Dudnikov[1] that has the remarkable capability of producing negative hydrogen ion current densities up to 3.7 amperes cm^{-2}. The discharge is maintained in an atmosphere of hydrogen (10^{16} molecules cm^{-3}) and cesium (10^{12} atoms cm^{-3}) with a tungsten cathode surface temperature of 600-800°K. Under these conditions something less than a monolayer of cesium forms on the cathode surface. From the H^- charge exchange rate one derives an H atom density of 10^{15} atoms cm^{-3}. Consideration of H^- collisional destruction processes leads to the conclusion that the electron density is $1 - 3 \times 10^{14}$ electrons cm^{-3}, with an electron temperature range 3 – 10 eV. The energy distribution of H atoms bombarding the cathode ranges from thermal energies up to 10 eV; the incident atomic flux with energies greater than a few eV is perhaps as large as 20 amperes cm^{-2}. Both the proton flux and the atomic flux will contribute to the negative ion production. A mechanism for negative ion production by atomic bombardment of the cathode via the intermediary of a CsH molecular formation is described.

*Work performed under the auspices of the U.S. Energy R. & D. Administration.

[1] Y. I. Belchenko, G. I. Dimov and V. G. Dudnikov, Nuclear Fusion 14, 113 (1974).

Prospects and Problems of Colliding-Beam Tokamak Plasmas*

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A colliding-beam tokamak plasma (CBT) consists of space-charge neutralized ion beams (set up by neutral injection) travelling parallel and anti-parallel to B_t.[1] The energetic ions are removed from the plasma before completely thermalizing on the electrons. With oppositely directed D and T beams, the fusion gain Q is somewhat larger than in a TCT system.[1] We show that the fusion power density P_f can be comparable to that in the TCT, but the CBT plasma must be smaller to permit adequate penetration by the lower energy beams (50-60 keV).

The results of Ref. 1 are modified when the toroidal electric field E_t is taken into account. For long-pulse operation, however, the CBT possesses the advantages of "automatic" refueling, and ability to maintain the plasma current indefinitely by means of the injected beams, with $E_t = 0$. For constant electron drift velocity, the condition on the initial injection energies is approximately $W_{To} \approx (n_D/n_T)^2 1.5 (W_{Do}^{1/2} - T_e^{1/2}/4)^2$. A small ripple in B_t can introduce the required "Z"> 1 without causing loss of parallel travelling ions.

Unavoidable charge-exchange loss of fast ions on the injected neutral beams reduces Q by 5-15%. Impurity-induced angular scattering compels an increase in injection energy, with some loss in Q and P_f. While techniques for the prompt removal of decelerated ions are conceivable, it seems likely that practical systems will operate in a CBT/TCT transition regime with $n_{beam}/n_e \gtrsim 1/2$. In fact colliding-beam fusion reactions can exceed beam-target reactions in present-day injection experiments in D-D, with $n_{beam}/n_e \sim 0.1$.

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[1] R.M. Kulsrud and D.L. Jassby, PPPL Report MATT-1114 (January 1975).

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ABSTRACT

PLASMA ROTATION IN A STRAIGHT θ-PINCH

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The previously unexplained plasma rotation observed in open-ended θ-pinch configurations is shown to result from the shorting out of electric field components at the insulating boundary.

A single fluid MHD model is employed with a small Hall term obtained by neglecting election inertia. An equilibrium initial state is assumed as well as an idealized geometry with insulating ends. At t = 0 the tangential electric field components are shorted at the insulator. This shorting propagates into the plasma, producing rotation.

Most of the rotational energy is propagated by Alfvén waves. However, small amplitude pre- and post-cursors are also observed. These are dispersive, with spacially and temporally modulated amplitudes and frequencies. The amplitudes and frequencies scale with the 3/2 power and inverse power of the strength of the Hall term, respectively.

Investigations of the Coupling of Heat and Particle Transport in a Plasma

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Nonequilibrium statistical mechanics predicts the cross coupling of various transport processes for systems not too far from thermal equilibrium. Recent computer simulations of plasmas have shown that both plasma and heat transport can be anomalous. The question naturally arises as to whether or not the cross-coupling is also anomalous. We have started investigations along this line using two dimensional computer simulation of plasma. We first modified an existing two-dimensional electrostatic particle model for a plasma in a constant magnetic field so as to have two boundaries at constant but different temperatures. We start the plasma at a temperature T_0, the boundary at $\chi = 0$ at $1.5\ T_0$ and the boundary at $\chi = L$ at T_0. The initial plasma parameters were, $n\lambda_D^2 = 1$, $\omega_{pe} = 3\omega_{ce}$, $m_i/m_e = 1.25$. We observe that the temperature profile quickly relaxes to its steady state value, at least an order of magnitude faster than can be accounted for by collisional heat diffusion. The density profile gradually distorts so as to go towards a constant pressure profile showing the expected coupling of heat and particle transport. The time scale for this is the same as for plasma diffusion which is almost the classical collisional value for the parameters chosen. A strong anomalous heat transport is observed which is consistent with the rapid relaxation of the temperature profile. We show that this transport is associated with long wavelength waves, $k\lambda_D < 1$, by turning off the electric fields of these waves and observing that the heat transport drops to a value near the collisional value. This causes only a slight decrease in the particle transport, showing that the heat transport is not associated with convective plasma motion. It appears that the transport must be due to upper hybrid waves but a detailed analysis is difficult for this set of parameters and has not been carried out; we had expected to find only the classical result here.

Work supported by NSF MPS74-15233 and ERDA AT(04-3)-34 P.A. 157.

The Expansion of a Narrow Slab of Plasma Across a Fixed Magnetic Field

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A crucial problem in plasma physics is the rate of diffusion of electrons across a magnetic field so as to neutralize charge build-up due to ions. This will influence whether or not plasma columns which are smaller than an ion larmor radius can be confined; it will determine electric field effects on the orbits of α particles and may modify the dispersion relations for waves involving substantial ion charge accumulation. Indeed recent computer calculations have shown that anomalous electron transport can significantly modify the damping of lower hybrid waves. Numerous experiments have also indicated that the electron transport can be anomalously large. To investigate this problem we have simulated the expansion of a thin slab of plasma across a magnetic field. The initial dimensions of the slab were small compared to the ion larmor radius but the electron larmor radius was much smaller than the slab (mass ratio 400 was used). The system remains stable until the ions move out past the electrons (the electrons expand one electron larmor radius beyond the initial slab edges very quickly and the ions must overtake them). Once the ions overtake the electrons the electrons become unstable to what appears to be a diochotron instability which transports the electrons across the field at the ion velocity. An analysis of the instability is now being carried out.

Work supported by ERDA AT(04-3)-34 P.A. 157 and NSF MPS74-15233.

ABSTRACT

THE EFFECT OF DISSIPATION ON RESONANT ALFVÉN WAVE ABSORPTION

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Earlier calculations have shown that strong coupling with an external RF source is possible at low frequencies in inhomogeneous plasmas through the continuum of stable Alfvén waves. The calculations have been performed for arbitrary diffuse profiles in straight geometry using the ideal MHD and guiding center models. Recently, interest has been placed on the departure from ideality of the models by including resistivity, viscosity, and finite Larmor radius effects. We will discuss the results to date of an analysis in which a thin resistive layer is included in the ideal MHD model, in particular its effect on the continuum of stable eigenmodes and the modification of the externally excited RF absorption in the continuum. A major result is that the presence of small but finite resistivity has no effect on the rate of energy absorption. Furthermore the plasma displacement within the resistive layer is related to the Airy function and suggests that a possible mechanism for energy transfer within the layer is a standing wave propagating from the layer center to its extremities.

Neutral Hydrogen Density Measurements by Optical Scattering

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Resonant scattering of optical radiation is a potentially useful method of measuring the density, n_H, of neutral hydrogen (and certain impurities) in fusion plasmas. The method permits spatial resolution, not easily attainable with line-of-sight techniques such as charge-exchange or optical emission measurements, is specific to the species being studied, and is sensitive enough to detect even small concentrations of the resonant species. Our analysis incorporates a time-dependent multi-level collisional-radiative model[1] for examining the population of states in neutral hydrogen subject both to a fusion plasma environment and to the probing optical beam. Results for time scales, scattered sign. size, signal-to-noise ratios, and dependence of the effect on plasma dens: and temperature and on probing radiation intensities will be presented. Measurements of neutral temperature and magnetic field fluctuations also appear feasible.

Lyman-α radiation, 1215 Å, appears best suited for this application because most of the hydrogen will be in the ground state, because the cro section for resonant scattering is large [$\sigma \sim 4 \times 10^{-14}/T_H(ev)^{1/2}cm^2$ for a laser band width equal to the Doppler full half-width], and because competing processes which prevent re-emission of the absorbed photons are minimized for the $n = 1 \rightarrow n = 2$ transition. The multi-level model shows a saturation of the scattered signal for incident optical flux intensitie above ~ 100 kW/cm^2; at this saturation limit, the enhancement, ε, of the emission coefficient of L-α radiation above its natural background level can be approximated by $\varepsilon \approx 10^{16}/n_e$, for conditions in the range of $n_e = 10^{12}-10^{15}cm^{-3}$, T = 50 - 5000 eV. Analysis of signal-to-noise ratios and criteria for detectability of signals in realistic fusion application coupled to predictions for the expected minimum value[2] of n_H($\sim 10^7-10^8$ cm demonstrates that a L-α source which can generate a probing beam with an intensity approaching the saturation limit within the L-α Doppler line wi ($\Delta\lambda_D \simeq 0.12 \times T(ev)^{1/2}$Å) is required. These intensities are not attainabl with electrical spark[3] or laser-plasma[4,5] sources, but appear well within the capability of recent advances in optical frequency conversion and nonl laser technology.[6,7]

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1. V. Myerscough, unpublished calculations.
2. D. Duchs, H. Furth, and P. Rutherford, PPL Report TM-265 (1973).
3. P. Bogen, Y.T. Lie, and D. Rusbuldt, Atomkernergie 14, 217 (1972).
4. D.W. Koopman, Phys. Fluids 15, 1959 (1972).
5. C. Breton and R. Papoular, Euratom Report EUR-CEA-728 (Jan. 1974).
6. S.E. Harris, Phys. Rev. Letters 31, 341 (1973).
7. R.T. Hodgsen, et. al., Phys. Rev. Letters 32, 343 (1974).

Renormalization Approaches to Anomalous Convective Transport*

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It has been suggested that convective transport due to low frequency, long wavelength thermal fluctuations may be responsible for the anomalous transport observed in various devices. We have developed a theory of anomalous viscosity caused by these fluctuations, using three related renormalization techniques; all predict:

$$\mu^2(k,s) = \frac{1}{2}(2\pi n\lambda_D^2)^{-1}(1+\Omega^2/\omega_p^2)^{-1}\ell n\left(\frac{s+2\mu k_c^2}{s+2\mu k^2}\right); \quad k_c \cong \min(\lambda_D^{-1}, r_c^{-1}).$$

The frequency - wavenumber dependence (i.e., non-Markovian behavior) arises from the long-lived nature of the modes, which themselves influence the transport (renormalization). The magnetic field scaling is in agreement with computer experiments.

All versions of the theory proceed from the Kubo formula $\mu = (mT)^{-1} \times \langle mv_x(t)v_y(t) \mid mv_x v_y\rangle$. In the Landau-Placzek method, the Kubo expectation is evaluated from a transition probability Γ which is assumed to be locally Maxwellian for long times. In the Generalized Langevin Equation approach, the propagation of bilinear variables (describing hydrodynamic mode coupling) is described explicitly; the self-consistent coupling of these variables gives the anomalous transport. In the kinetic theory approach, we consider a cumulant hierarchy of transition probabilities. By selecting a certain subset of all terms at each order, we generate a renormalized equation which we then solve for the required Γ. This equation retains all appropriate conservation laws and is thus much better than the diffusing orbit approximation, which leads in this case to erroneous results. The equation is closely related to the direct interaction approximation, and also follows simply from the general renormalization theory of Martin, Siggia, and Rose.[1]

It is expected that these techniques will be useful in more general situations, for example the anomalous transport due to hybrid waves.

[1] Martin, Siggia, and Rose, Phys. Review A8, 423 (1973).

† National Science Foundation Pre-Doctoral Fellow.

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The Neutralized Colliding Beam Torus*

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We consider the performance of a charge neutralized toroidal reactor consisting of "warm electrons" and energetic ions ($n_h = n_e$) injected to maximize head-on collisions.[1] There are no cold ions. The fast ions thermalize on the electrons and are removed before angular scattering is significant. The fusion energy gain is somewhat larger than the TCT, the $n\tau$ value is smaller, while only one-fourth to one-third the beam voltage is required.

[1] R. M. Kulsrud and D. J. Jassby PPPL Report MATT 1114 (1975).

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Particle and Energy Transport Due to Drift Waves in a Sheared Magnetic Field*

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A two-and-a-half dimensional bounded slab model has been developed to study particle and energy transport due to low-frequency drift waves in a sheared magnetic field. Instabilities caused by the universal modes have been observed along with the enhanced cross-field diffusion for the long-wavelength modes in a collisionless low-β plasma. The effect of the sheared magnetic field on the instability has been investigated by varying the strength of the shear. Results for the critical shear for the onset of instability are shown to be consistent with the Pearlstein and Berk criterion which predicts a rather strong shear that may not always be available in a tokamak. For the case with a moderate shear, the growth rate is substantially reduced, however, the instability is found to cause large diffusion eventually. In addition to the particle diffusion, we have also observed some ion heating associated with the energy tranfer of the drift waves. The details of the mode structure and the scaling of the energy and particle transport will be reported.

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Anomalous Heat Transport Across A Magnetic Field

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A self-consistent theory of heat transport across a strong magnetic field for a plasma near thermal equilibrium has been developed taking into account the effects of wave convection. It is shown that when the mean absorption length of the plasma waves is small compared with the size of the system, the heat transport can be described by a diffusion process. The electron thermal conductivity coefficient thus obtained is directly proportional to the electron temperature in the low temperature region (where the absorptions of the long wavelength modes are predominantly collisional). Whereas in the high temperature region where strong electron cross-field diffusion becomes important, the conductivity no longer increases with the increase of the electron temperature. Instead it saturates to a value which is independent of the temperature but is directly proportional to the magnetic field strength. The limit when the mean absorption length of the waves is no longer small compared with the length of the system is also discussed.

Work supported by NSF, ERDA and ONR.

STABILITY ANALYSIS OF SHARP-BOUNDARY VLASOV-FLUID SCREW-PINCH EQUILIBRIA

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Using the Vlasov-fluid model[1] (collisionless ions, massless electrons, quasi-neutrality, displacement current neglected, $\omega \ll \omega_{pi}$), we have derived a dispersion matrix for sharp-boundary screw-pinch equilibria in which the plasma is separated from a conducting wall by a vacuum region. Because of the kinetic description of the ions, one cannot use pressure balance as one of the boundary conditions on the magnetic field, although pressure balance is an exact consequence of the model equations. The appropriate boundary conditions for the magnetic field at the plasma-vacuum interface are that the current normal to the interface be zero and that the normal component of the magnetic field be continuous.

During the evolution of a sharp-boundary screw pinch, the boundary moves bodily and deforms, thereby affecting the dynamics of the collisionless ions. We have taken these boundary motions into account by introducing a time-dependent transformation of the phase space to a set of variables in terms of which the time-dependent perturbed boundary appears like a rigid cylinder at rest. Writing the collisionless Boltzmann equation and the $(\nabla \times \underset{\sim}{B}) \times \underset{\sim}{B}$ equation in terms of the new variables, we then consider perturbations about a state which is an equilibrium modified by some time-dependent first-order effects of the boundary motions. Formally, in terms of the new variables, the equations for the linearized problem are the same as those that would describe perturbations with a hypothetical, rigid, motionless, cylindrical boundary, except that a boundary term must be added to the collisionless Boltzmann equation.

The function ξ, defined by $A^{(1)} = \xi \times B^{(0)}$, is expanded in terms of eigenfunctions ξ_{ℓ} of an operator $\underset{\sim}{F}$, where $\underset{\sim}{F}$ is the same as the MHD force operator, except that the term proportional to the adiabatic compressibility γ is absent. An auxiliary function, closely related to the perturbed ion distribution function, is expanded in terms of eigenfunctions of the unperturbed Liouville operator. The dynamics of the system is characterized by an infinite-dimensional dispersion matrix whose rows and columns are labelled by the functions ξ_{ℓ}. If the expansions are truncated by keeping only the first M Liouville eigenfunctions and N+1 functions ξ_{ℓ}, then the Vlasov-fluid eigenfrequencies for the sharp-boundary screw pinch are approximated by the zeros of a polynominal of degree M+1. In the zero-ion-temperature limit with the plasma β held constant, we can show that there are two nontrivial eigenfrequencies. For N=0, which corresponds to keeping only one eigenfunction of $\underset{\sim}{F}$, these nontrivial eigenfrequencies are identical to the eigenfrequencies of incompressible ideal MHD for the sharp-boundary screw pinch.

This formulation will be evaluated numerically by using techniques described earlier[2] for evaluating the Vlasov-fluid model with diffuse equilibrium profiles. Our goal is to understand kinetic effects that are absent in ideal MHD.

[1] J. P. Freidberg, Phys. Fluids 15, 1102 (1972).

[2] H. R. Lewis and J. P. Freidberg, Fifth European Conference on Controlled Fusion and Plasma Physics (Grenoble, France, August 21-25, 1972), paper 41.

Anomalous Resistance for A Beam - Plasma System with Return Current

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Abstract

Of the several mechanisms proposed for plasma heating using intense relativistic beams, one of the most promising involves the large amplitude electrostatic fields near the plasma frequency arising from the two stream instability. The coupling of these waves to the plasma can be greatly affected by the presence of the return current required to provide current neutralization for beams exceeding the Alfvén-Lawson critical current, and these oscillations can also give rise to an anomalous resistivity for this current. We have carried out computer simulations to investigate the time evolution of a beam-plasma system which also includes the return current. The results show that at early stages of the parametric instability the beam mode decays into a backward ion wave and a forward plasma wave with considerably shorter wavelength. This results in the formation of a non-symmetric velocity distribution of the plasma particles. The presence of the beam-plasma instability enhances the resistivity for the return current substantially. At later stage the parametric instability dissipates the return current and generates a net plasma current in the direction of beam propagation.

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THE ANIMAL CODE*

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ABSTRACT

Introduced in this presentation is the computer code ANIMAL - A New Implicit Magnetohydrodynamic ALgorithm. ANIMAL is a descendant of the code originally developed to study theta pinches and laser produced plamas [1] and has most recently been used to study toroidal theta pinches and sausage instabilities of straight Z pinches. ANIMAL solves time-dependent, one- and two-dimensional Eulerian MHD equations using alternating-direction implicit finite difference equations. The equations are solved in rectangular, cylindrical, and toroidal coordinate systems. A moving grid which expands or contracts at a prescribed rate is used to calculate accurately situations where length scales change greatly. To follow "preionization" discharges through several cycles, improved numerical methods [2,3] and boundary conditions [3] have been developed, and, equations of state which take into account ionization, bound-free and bound-bound radiation and electron-neutral resistivity have been incorporated into the code in addition to the usual electron-ion resistivity and ion and electron thermal conduction. Sample calculations are presented.

REFERENCES

1. Lindemuth, I., and Killeen, J., J. Comp. Phys. 13, 181 (1973).

2. Lindemuth, I., "Conservation Form in Computational Magnetohydrodynamics", accepted for publication in Journal of Computational Physics.

3. Lindemuth, I., "Numerical Methods in the ANIMAL Code", submitted for presentation at Seventh Conference on Numerical Simulation of Plasmas, Courant Institute, N.Y., N.Y., June 1975.

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Abstract Submitted to

Sherwood Theoretical Meeting

Washington, D. C.

Plasma Turbulence Due to Runaway Electrons in Tokamaks

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Plasma waves in a magnetic field are shown to be unstable to runaway electrons, once a critical number of runaways is exceeded. The runaway electrons in turn are scattered in pitch angle by the turbulent plasma waves. The perpendicular energy of the runaway electrons is therefore increased and the synchrotron emission is enhanced. The enhanced emission calculated from the quasilinear, steady distribution function is to be compared with the experimental observation.

LOW-FREQUENCY STABILITY OF HIGH-CURRENT PARTICLE RINGS

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Abstract

A study is made of the low-frequency magneto-hydrodynamic stability of high-current particle rings. The directed kinetic energy-density of the current carrying particles or beam is allowed to be a non-negligible fraction of the energy-density of the external magnetic field. The analysis is valid for relativistic and non-relativistic beams. Two configurations are investigated in detail: astron geometry where there is no toroidal magnetic field, and tokamak geometry where a strong toroidal field exists. For both configurations an energy principle is derived. For an astron, a sufficient condition for stability of the kink modes ($m = \pm 1$, $n \neq 0$) is found to be $\lambda_\beta > 2\pi R$, where λ_β is the self-field betatron wavelength of the beam particles, R is the major radius of the torus, and m (n) is the mode number for the minor (major) circumference. For a tokamak, it is found that stability conditions for the modes ($m \neq 0$, $n \neq 0$), with $n^2 \ll (R/a)^2 m^2$, are the same as the stability conditions for these modes in a corresponding conventional plasma pinch. (R/a is the aspect ratio of the torus.) The correspondence is given by $p \rightarrow (\frac{1}{2} n_B M_B u_z^2)$, where $p(r)$ is the isotropic pressure of the plasma pinch, and $(\frac{1}{2} n_B M_B u_z^2)$ is the directed kinetic energy-density of the particle beam. In particular, the usual Kruskal-Shafranov condition imposes an upper limit on the current in the particle ring.

Abstract for CTR Theory Meeting, April 7-9, 1975

NONLINEAR INITIAL MOTION OF A BELT PINCH

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The full nonlinear single-fluid dissipative MHD equations are solved numerically to study the initial pinching, bouncing, and shift of a toroidal belt pinch. The program is two-dimensional (poloidal plane) and time-dependent, and is similar to our previous screw pinch program. Both conducting shells and vertical fields are investigated, and the effect of the vertical field in reducing the toroidal shift is seen very clearly. The calculations typically are carried over two to three bounce periods. The calculated results are compared with the Garching belt pinch, and calculations are also made for our own projected experiment at Columbia.

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Equilibrium and Stability of Plasma-Neutral Layer in the Tokamak*

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Equilibrium and stability properties ($\omega \cdot r \cdot t$ to low-frequency drift waves, $\omega/\Omega_i \ll 1$, where Ω_i is the ion-cyclotron frequency) of the neutral-plasma layer near the boundary of a tokamak are studied under two different sets of condition:

(a) Ion-neutral charge exchange collisions are infrequent; therefore only the ionization collisions determine the neutral density profile.

(b) Ion-neutral charge exchange collisions are large, and the neutrals can be treated within the framework of fluid equations.

Where the effective cross-field diffusion coefficient is independent of the density of the plasma, analytic expressions for the neutral and plasma density profiles are calculated in both cases. The stability properties of drift waves are studied by perturbing the system and it is shown that the drift waves are damped in both cases, diffusion providing the stabilizing mechanism for the former and charge exchange collision provides the corresponding mechanism for the latter.

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Stabilization of the Dissipative Trapped Electron

Instability by Mode Coupling

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The dissipative trapped electron mode instability in a tokomak is basically a drift wave destabilized by a small fraction of trapped electrons. If finite ion inertia and finite larmor radius effects are neglected, one can show that the nonlinear equation describing the drift wave is

$$\frac{\partial \phi}{\partial t} - V_D \frac{\partial \varphi}{\partial y} - NV_D \varphi \frac{\partial \varphi}{\partial y} = 0 \qquad (1)$$

where φ is the perturbed potential normalized to Te/e, V_D is the electron diamagnetic drift velocity and N is the ratio of density to temperature gradient scale lengths.

The presence of trapped electrons, shear, finite ion larmor radius and inertia destabilizes modes at low wave number, provides damping at high wave number and lowers the frequency from the drift wave frequency. The nonlinear term in Eq. (1) couples the unstable waves at low wave number to damped waves at high wave number so the system evolves to a steady basically coherent, state. We find this nonlinear steady state computationally and can then calculate particle and energy loss rates for typical tokomak parameters in a form suitable for inclusion into diffusive timescale transport codes.

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Straight $\ell = 1$ Helical Equilibrium and Stability+)

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+) This work was performed as part of the joint research programme between Institut für Plasmaphysik, Garching, and EURATOM.

The equilibrium and stability of a straight, diffuse, two dimensional $\ell = 1$ configuration are investigated. The equilibrium depends of four parameters: β; δ, the helical plasma distortion; b/a, the ratio of wall radius to plasma radius; and ha, the helical wave number. All four are arbitrary. It appears that a limitation similar to Freidberg's $\beta\delta < \pi^2/16$ (1) can be expected. The restriction is, however, much less severe and equilibria have been obtained almost an order of magnitude past this limit. This condition arises from non linear effects and not because of bifurcation(2).

The stability of the numerically obtained equilibria is investigated using a δW analysis(3). The perturbation is expanded in a double Fourier series with unknown coefficients. Only ha is assumed to be small. We investigate modes with sinusoidal dependence $m\theta + kz$ times a function with the same symmetry as the equilibrium. The main result is that for m = 1, k = 0, the plasma remains stable even for $\delta = 2$. The frequency is given, to within a factor of about 1.2, by the sharp boundary formula $\omega = h v_A \delta \beta a^2/b^2$. The absence of the destabilizing effect of large δ is consistent with the work of Weitzner(4). It is, however, surprising that it persists for such large values of δ.

For the m = 1 mode it is also observed that the least stable mode is not necessarily for k = 0, although it is very close to this. The m = 2 mode is found to be unstable as sharp boundary theory predicts.

References

(1) J.P. Freidberg, Phys. Fl. 14, (1971)
(2) B. Marder and H. Weitzner, Plasma Physics 12, (1970)
(3) J.P. Freidberg and B. Marder, Phys. Fl. 16, (1973)
(4) H. Weitzner, N.Y.U., MF-76, COO - 3077 - 14, (1972)

ABSTRACT

Annual Meeting on Theoretical Aspects of Controlled Thermonuclear Research.
Washington, D. C., April 7-9, 1975

Energy Multiplication Studies for a Steady State Two Component Torus

by K. D. Marx, J. Killeen, A. A. Mirin and M. E. Rensink

Steady-state distributions are computed for a two component torus, using a two-dimensional (in velocity space) multispecies Fokker-Planck code. The species involved are electrons, deuterium (with a beam source term at average energy E_b), tritium, and oxygen as an impurity. At steady state there is equilibrium between the beam current and a deuterium loss term which models collisional losses, and equilibrium between beam input energy and deuterium and tritium collisional energy losses, coupled with energy exchange between species.

Parameters which are varied are beam energy E_b (over the range 100-200 KeV), beam current J_b, and the impurity concentration. The dependence of Q (the ratio of thermonuclear power to injected power) on these parameters is determined.

The significant quantity determining the loss rate of thermalized deuterium is the temperature of the background plasma. Since the heating of the background plasma increases with increasing J_b, and the rate of thermalization of the beam decreases with increasing E_b, Q is strongly dependent on these two parameters. However, it is found that, for fixed input power $J_b E_b$, Q is insensitive to variations in E_b from 100 KeV to 200 KeV.

High Frequency Convective Instability in the Skin Phase of a Tokamak

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The early state of a strongly turbulent heated magnetized plasma when the current is still confined in the skin is studied for two different types of the stream velocity profile. The geometry is that of an inhomogeneous magnetized plasma slab which supports a stream v(x) in the z-direction with $v(x) \geqslant v_{the}$, and $\omega_{ce} \geqslant \omega_{pe}$, $d \ln n_o / d \ln v << 1$, $d \ln T_e / d \ln v < 1$. The poloidal β of order 1 and the poloidal magnetic field scale length of the order of few collisionless skin depth. For the velocity profile we consider two types: 1) when the velocity profile is a smooth function of x and 2) when the velocity profile has an abrupt change over a thickness smaller than c/ω_{pe}. For $\omega/k_z > v$, $\omega/k_z > \omega_{pe}/k_y$ then the ratio of growth rates $\gamma_1/\gamma_2 \simeq (L k_z)^{-1} (2k_z v_{th}/\omega_{ce})^{\frac{1}{2}}$ with L the velocity profile scale length. For typical (TTT) parameters we have $\gamma_2 > \gamma_1 \sim \omega_{pe}$ and the frequencies are of the order $k_z v(x)$. When use is made of the quasilinear approach with electron trapping as the saturation mechanism an effective resistivity of the order or larger than the Buneman is obtained. For the waves that propagate at oblique angles linear wave conversion at the plasma boundary implies radiation with a frequency much larger than ω_{pi}.

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Numerical Simulation of Trapped-Electron Instability*

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We have studied the dissipative trapped electron instability and the associated anomalous transport processes using a 2 1/2 - D particle simulation model. Monte Carlo collision operator was used to introduce small-angle Coulomb scattering, and a square mirror magnetic field was assumed to trap electrons. The plasma is bounded in x-direction (radial direction), electric potential being fixed at the boundaries, and is periodic in y-direction (azimuthal direction). Initial results show that the most unstable mode grows in time and saturates, accompanied by the enhanced particle diffusion and heat transport. The measurements indicate that the electron thermal conduction is larger than the electron diffusion by a factor of 2-3. In addition, it was observed that particles are scattered out of the mirror at the time of saturation of the instability. The potential is seen to be peaked at the center of the mirror in the y-direction.

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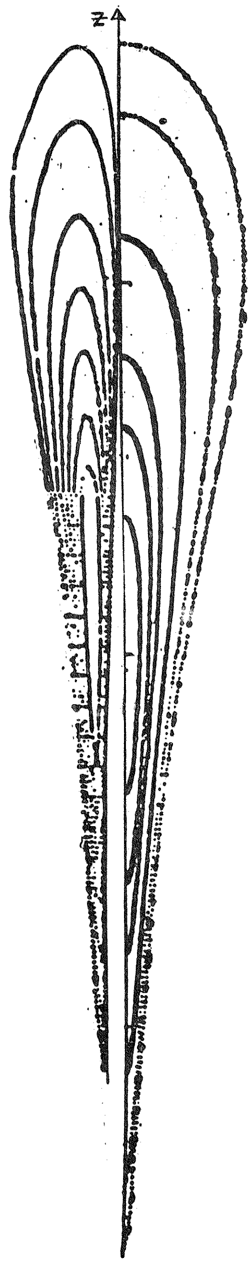

THREE-DIMENSIONAL FLUX TUBE EQUILIBRIA IN 2XIIB

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The guiding center fluid equations have been solved numerically according to the scheme given by Hall and McNamara[1], for 2XIIB and Baseball equilibria of the form $\underline{\underline{p}} = \omega(\bar{\Gamma})\ \hat{\underline{\underline{p}}}(B)$, and preliminary results are available. The plasma loading function ω is specified at the mid-plane by its variation along a single ray in a symmetry plane of the field, i.e., $\omega = (1 - x^2/a^2)^2$ at $y = 0$, $z = 0$. The plasma pressure surfaces, $\bar{\Gamma}$, are determined by solving the magnetic differential equation, $\nabla\cdot(\Gamma \underset{\sim}{B}) = \hat{p}_{\parallel}$, giving $\bar{\Gamma} = \oint d\ell \hat{p}_{\parallel}/B$ on each field line. The magnetic induction, including plasma, is given by $\underset{\sim}{B} = \underset{\sim}{B}_V - \nabla\Omega + 4\pi(\underset{\sim}{M}_\nu + \underset{\sim}{M}_J)$ where $\underset{\sim}{B}_V$ is the vacuum field of the coils and Ω is the magnetic scalar potential determined by $\nabla\cdot\underset{\sim}{B} = 0$. The gyromagnetization is $\underset{\sim}{M}_\nu = (p_\perp - p_\parallel)\underset{\sim}{B}/B^2$ and the drift magnetization is $M_J = -\chi\nabla(\Gamma/\bar{\Gamma})$, where $\chi = \int \bar{\Gamma} d\omega$. A contour plot of the gyromagnetization on the symmetry plane y=0 of 2XIIB is shown on the right of the figure. On the left, $\hat{\underset{\sim}{b}}\cdot\underset{\sim}{M}_J$ is given, but with considerably reduced scale heights. The implication of the results for the construction of high β linear magnetic wells will be discussed.

*Work performed under the auspices of the U. S. Energy R. & D. Administration.

[1] L. S. Hall, B. McNamara, Lawrence Livermore Lab. Rpt. UCRL-75774 (1974). To be published in Phys. Fluids.

ABSTRACT

Annual Meeting on Theoretical Aspects of Controlled Thermonuclear Research
Washington, D.C., April 7-9, 1975

Energy Multiplication Studies for a Pulsed Two Component Torus

by A.A. Mirin, J. Killeen, K.D. Marx and M.E. Rensink

Energy multiplication studies for a pulsed two component torus are performed using a time dependent two-dimensional multi-species Fokker-Planck code. The time derivative $\partial f_a/\partial t$ of the distribution function f_a is written as the sum of a Fokker-Planck collision term, a source term, a major radius compression term and a term expressing finite particle and energy confinement times.

In the scenario presented a deuterium heating beam is injected into a cold tritium plasma. After thermalization has occurred, a deuterium fusion beam is injected, and this is followed by major radius compression. At this time, t_i , the breakeven experiment begins, and lasts until the resulting plasma has cooled back down to its temperature at time t_i.

Calculations are performed with and without impurities. We vary such parameters as the injection energy, compression factor, compression time and the composition of the heating beam (e.g. deuterium or tritium). Output diagnostics include Q (the ratio of the thermonuclear power to the injected power).

Investigation of a P-B^{11} Fuel Cycle for an Advanced CTR Utilizing Magnetic Confinement

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The advantages of the $B^{11}(P, 2\alpha)H_e^4$ reaction as an energy source for a CTR include negligible production of high energy neutrons and potentially high recovery efficiencies for the charged particle kinetic energy released. High Bremsstrahlung emission rates pose a problem for steady state operation, so a pulsed cycle has been investigated. Equations governing the time development of the reaction product velocity distribution, the energy production, Bremsstrahlung emission and energy transfer among Maxwellian and non-Maxwellian plasma components have been derived and integrated numerically. Time histories of relevant plasma parameters have been obtained for conditions which nearly optimize energy yield against Bremsstrahlung. For periods greater than the energy doubling time the average rate of energy yield exceeds the average Bremsstrahlung emission rate. Cyclotron radiation is, of course, also important for the overall energy balance and its effect depends upon geometric considerations as well as reflectivity of the walls.

This work was supported by the U. S. Atomic Energy Commission Contract No. AT-(40-1)-4478 and by the Electric Power Research Institute.

Spiky Turbulence: Its Generation and Effect on Plasmas

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It has been demonstrated recently[1] that electric fields of high intensity can be sharply localized within the density cavities produced by the ponderomotive force of high frequency waves. Thus far these localized fields have been studied mainly in the context of the interaction of electromagnetic radiation with plasmas, where they have been found to play a dominant role. In the present investigation we study the possibility of triggering the generation of such localized fields by a convectively unstable electron plasma wave, as might be the case when a fast beam is injected into a plasma. In this situation it is found that a new turbulent state consisting of a collection of neighboring spiky fields is generated. This spiky turbulence grows in time from an initial noise wave having the same wavelength as the pump, but moving in the opposite direction. The interaction of individual electrons with localized fields has been found to be well described by the Born-scattering approximation. We use this approximation to calculate[2] the effect of the spiky turbulence on the electron velocity distribution function. The results of this theoretical model are compared with a recent beam-plasma experiment at UCLA, in which the existence of this new type of turbulence has been observed.

[1] G.J. Morales, APS Bull. 20, 39 (1975).
[2] G.J. Morales and Y.C. Lee, Phys. Rev. Lett. 33, 1534 (1974).
Work supported by ERDA, ONR and NSF.

A Diffusion Theory for the Interaction of Relativistic Electron Beams with Fusion Target Blow-Off Plasmas*

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A formalism is developed to model the interaction of a relativistic electron beam with high-atomic-number plasmas blown off of fusion targets. The relativistic Boltzmann equation with a Fokker-Planck collision term is solved with the assumption that elastic scattering characterizes the shortest time and length scales of the beam-plasma system. The general solution for $f(\vec{p}, \vec{x}, t)$ incorporates arbitrary beam-plasma configurations and electromagnetic fields, and contains energy-dependent dynamic friction. Explicit one-dimensional solutions containing the self-consistent return-current electric field show that although range-shortening occurs, the primary effect of the electric field is to prevent high-current-density beams from penetrating into the dense-plasma region.

*Work supported by the Defense Nuclear Agency.

MHD STABILITY AND BETA LIMITATIONS OF THE ELMO BUMPY TORUS*

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ABSTRACT

In this paper the MHD stability of the ELMO Bumpy Torus (EBT) is analyzed. The EBT is comprised of a set of twenty four linked mirrors, arranged to form a torus, and heated by microwave RF energy. The resulting plasma has two basic components, the first consisting of mirror confined, high beta, very hot electron plasmas, which form hollow annuli between each pair of coils; the second is a moderate temperature toroidal ion plasma which threads each of the electron annuli.

Although the annuli are predicted to be unstable according to the guiding center theory, the Larmor radii of this component are too large for the guiding center model to be valid. Using the appropriate scaling of the Vlasov equation we find evidence that the high drift frequency of the relativistic electrons suppresses instabilities at frequencies below ion gyrofrequency in the annuli.[1] The experiment likewise evinces complete stability of the annular component.

Accordingly we consider the high β annuli to be rigid, noninteracting (on the stability time scale) "invisible windings" and seek stability boundaries and β limitations of the toroidal component. The configuration is modeled by an axisymmetric bumpy cylinder with identified ends. The coupled nonlinear 2D guiding center equilibrium equations are solved self-consistently for high β anisotropic annuli with an isotropic core. A modified energy principle is used[2] and the least stable modes are identified as localized, pressure driven ballooning modes. However, the stabilizing effect of the annuli (mainly by creating a region of negative V'') allows maintenance of a toroidal β of at least twenty percent which is stable to all modes encompassed by the guiding center or ideal MHD theories.[3] There is indication that further analysis will reveal even higher stable beta.

*Research sponsored by Energy Research and Development Administration under contract with Union Carbide Corporation.

1 G. E. Guest, C. L. Hedrick, D. B. Nelson, to be published in Phys. Fl.
2 J. L. Johnson, R. M. Kulsrud, K. E. Weimer, Plasma Phys. 11, 463 (1969)
3 G. O. Spies, D. B. Nelson, Phys. Fl. 17, 1865 (1974)

BALLOONING-INTERCHANGE STABILITY BOUNDARY FOR LONG, THIN, AXISYMMETRIC, LINE-TIED, MIRROR TRAPS*

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The guiding-center energy principle has been used to calculate the maximum β consistent with stability (due to line-tying at the ends) against ballooning interchange modes. Self-consistent equilibria were computed assuming (1) large aspect ratio, and (2) axisymmetric geometry (purely poloidal field). The stability criterion was then obtained by numerical integration of the Euler-Lagrange equation resulting from the energy minimization. Results show a sensitivity to the z dependence of the vacuum field. Higher β's are permitted when the minimum in B at the midplane is very flat; this occurs because the bad-curvature regions are pushed out toward the ends, where the plasma pressure is low. For vacuum mirror ratios less than 3 the β limitations are tolerable.

*Work performed under the auspices of the U.S. Energy R. & D. Administration.

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Envelope Solitons and Envelope Shocks

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The present paper reviews the recent theoretical works conducted at Hirbshima University on stationary, localized high--frequency waves confined in self-consistent density profiles. The report covers the following subjects:

1) General one-dimensional envelope soliton solutions trapped in rarefactory acoustic solitons and their basic properties including stability against perpendicular perturbations.
2) One-dimensional BGK-like solutions for plasma wave trapped in ion density perturbations in the medium of untrapped plasmons.
3) One-dimensional envelope quasi-shock solutions with account of reflected electrons in the upstream region.
4) Long-range interactions among envelope solitons via untrapped particles in a one-dimensional plasma.
5) Two-dimensional and three-dimensional envelope solitons in a medium of saturable nonlinear susceptibility and their basic properties including their stability against localized perturbations and their damping due to transit particles.

Comparison of some of the results with recent experiment will also be added.

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EXPANSION OF TOROIDAL EQUILIBRIUM WITH SMOOTH PRESSURE PROFILE AND INFINITE SETS OF CORRUGATING FIELDS IN LEADING ORDER

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ABSTRACT

Expandable toroidal equilibria of the high-ß stellarator type (HBS, Scyllac) without longitudinal current are investigated. The expansion parameter is $r \cdot \kappa \sim \varepsilon$, where r is the plasma or wall radius, κ the lowest-order curvature of the magnetic axis; the plasma-ß, periodicity length L, and torsion τ of the magnetic axis in lowest order are scaled as $\beta \sim 1$, $2\pi r/L \sim 1$, $r\tau \sim 1$. An arbitrary smooth pressure profile is considered and the second order solubility condition, which represents the condition for toroidal equilibrium in leading (i.e. second) order, is (formally) solved by admitting infinite sets of first order corrugations. The sets corresponding to the case of an $\ell = 0, 1$ surface current (Scyllac) equilibrium are

$$\Theta + \zeta\,,\quad 3\Theta + 3\zeta\,,\ \ldots,\quad n\Theta + n\zeta\,,\ \ldots$$

$$\zeta\,,\quad 2\Theta + 3\zeta\,,\ \ldots,\quad (n-1)\Theta + n\zeta\,,\ \ldots$$

where Θ is the Hamada coordinate describing the coordinate the short way around and ζ the Hamada-like coordinate increasing by unity per period the long way around. The sets corresponding to the case of the $\ell = 1, 2$ surface current (HBS) equilibrium are

$$\Theta + \zeta\,,\quad 2\Theta + 2\zeta\,,\ \ldots,\quad n\Theta + n\zeta\,,\ \ldots$$

$$2\Theta + \zeta\,,\quad 3\Theta + 2\zeta\,,\ \ldots,\quad (n+1)\Theta + n\zeta\,,\ \ldots$$

In each case the solution of the second-order condition determines the products of the amplitudes of corresponding pairs of the two sets. For a finite number of pairs the amplitudes are determined by numerically minimizing the deviation from toroidal equilibrium in second order. For a smooth profile which is not close to the surface current (flat) profile but, for example, nearly parabolic, it turns out that the single-pair approximation is rather poor; including the second pair already leads to a significant improvement of toroidal equilibrium. The amplitudes of the corrugations are calculated for various ß values and shapes of the profile.

"This work was performed under the terms of the agreement on association between the Max-Planck-Institut für Plasmaphysik and EURATOM".

The Frequency and Damping of Ion Waves*

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We have numerically solved the linearized Fokker-Planck equation for the frequency and damping rates of ion-acoustic waves in the presence of collisions. The electrons have been treated as an isothermal fluid. The cases $T_e/T_i = 0.5$, 1, 2, and 4 have been treated. The results agree well with known results for strong and weak collisions.

* Work supported jointly by U.S.E.R.D.A. Contract AT(11-1)-3073 and U.S.A.F.O.S.R. Contract F 44620-70-C-0033.

Minimum Energy Reverse Field Configurations

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Recently the theory developed by J. B. Taylor[1] has predicted the relaxation of a stabilized z-pinch configuration to a minimum energy state which, if the pinch ratio $\Theta = B_\theta(r_{wall})/B_{zaverage}$ is larger than 1.2, is characterized by generation of reverse axial magnetic field in the outer plasma region. The theory is for the $\beta = 0$ case, but leads to configurations which are close to high-beta stable ones;[2] these theoretical results together with experimental evidences indicate that, under certain conditions the plasma moves by itself ("self-reversal") to a reverse field configuration without programming. A simple calculation[3] will be presented, based on the sharp-boundary model, which allows to derive some interesting features of the cylindrical MHD equilibrium with finite beta. The plasma is assumed to be enclosed in a rigid, perfectly conducting cylindrical vessel. Then for a given axial flux, we minimize the potential energy with respect to $Z = B_z(\text{wall})/B_{zaverage}$ for given values of β_θ and Θ. In particular minimum energy equilibria characterized by a reversed axial magnetic field appear when Θ exceeds a critical value which depends on β_θ. The $\beta_\theta = 0$ case leads to result which are close to those of the Taylor's theory as far as the cylindrical symmetric case is concerned.

* On leave of absence from Centro di Studio sui Gas Ionizzati del Consiglio Nazionale delle Ricerche e dell'Universita di Padova (Associazione Euratom-CNR) - Padova (Italy)

1. J. B. Taylor, Fifth Conference on Plasma Physics and Controlled Nuclear Fusion Research, Tokyo, 11-15 November 1974, IAEA-CN-33/Post Deadline Paper.
2. E. P. Butt et al, Fifth Conference on Plasma Physics and Controlled Nuclear Fusion Research, Tokyo, 11-15 November 1974, IAEA-CN-33/E9-2
3. S. Ortolani, Rostagni G., University of Padua report to be published.

ABSTRACT

MHD CONTINUOUS SPECTRUM IN TOKAMAK GEOMETRY

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The linear MHD equations have discrete normal modes as well as a continuous spectrum. The continuous spectrum can lead to non-exponential growth or decay and is an equally important part in determining the plasma evolution. There has also been some recent interest in the possible use of the continuous spectrum for plasma heating because of its relative long wavelength and low frequency.

The present work considers the continuous spectrum problem in the toroidal Tokamak geometry. It is shown that the continuous spectrum is given by a reduced (one-dimensional, non-singular) eigenvalue problem for a system of four first order equations on the resonant flux surface.[1] Properties of the reduced eigenvalue problem are obtained, and solutions of the full set of linear MHD equations are constructed near the resonant surface. We show that one and only one solution is singular at the resonant surface while all other (infinitely many) solutions are non-singular. The singularity of the singular solution at the resonant flux surface is shown to be logarithmic in the flux variable.

[1] A similar system has been independently obtained by J.M. Greene.

A Transport Model with Moving Noncircular Flux Surfaces

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In the development of a toroidal transport code, which incorporates arbitrary, time-varying flux surface geometries, a number of logical and mathematical obstacles are encountered. These problems are studied in the context of a model with classical, electrical and thermal conductivities. Transport is envisioned as proceeding through a sequence of MHD equilibria. The implications of this discretization of the problem, which amounts to a decoupling of the transport and equilibrium equations, are examined numerically. A limiting time-independent solution is obtained.

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COMPUTATIONAL RESULTS ON MIRROR-CONFINED PLASMAS*

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Livermore, California

The effect of injecting several megawatts of neutral beam power into a mirror-confined target plasma in the 2XIIB experiment is studied with a two-dimensional velocity-space Fokker-Planck code. The individual behavior of both the target and the injected hot ion components in the plasma model is examined. Target decay in the absence of sources is compared with calculations using various injected beam currents. The effects of a background gas on both the target and the hot ion plasma are demonstrated. Magnetic compression of the target plasma during buildup is included in the model.

The essential characteristics of the plasma confinement system for a mirror reactor can be expressed quantitatively in terms of a few simple parameters. Traditionally, the power gain factor Q has been emphasized. However, power density (as measured by the plasma pressure parameter β) is also an important design consideration. Limits on β are imposed by the MHD stability conditions for the mirror and firehose modes. Values of Q and β_{max} are obtained from numerical solutions of the Fokker-Planck equations for D-T plasma systems. The trade-off between high Q and high β_{max} is examined in terms of the mirror ratio and the injection energy and angle of the ion sources.

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Abstract for CTR Theory Meeting, April 7-9, 1975

BIFURCATION OF MAGNETOHYDRODYNAMIC EQUILIBRIA

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The equilibrium (Grad-Shafranov) equation,

$$\Delta^* \Psi = -r^2 \frac{dP(\Psi)}{d\Psi} - \frac{d}{d\Psi}\left(\frac{f^2(\Psi)}{2}\right)$$

which results from the specialization of the magnetostatic equations to the case of an axisymmetric cylindrical (r,φ,z) coordinate system, is solved numerically in a domain of rectangular cross section (belt pinch) surrounded by a perfectly conducting shell. A simple algebraic "one point" approximation is shown to predict the occurrence of bifurcated (multiple) solutions to the boundary value problem over a wide range of parameters and choice of the functions $P(\Psi)$ and $f(\Psi)$. In particular, we approximate the sharp boundary model by rapidly changing but smooth profiles and vacuum regions are obtained by profiles which vanish for Ψ above a certain value. The numerical scheme is designed to catch all the bifurcated solutions which may occur for a single boundary value.

The dependence of the ratio of the major to minor axis of the plasma cross section on boundary value, parameter and function choice is discussed.

Work supported by U.S. ERDA under contract AT(11-1)-2456.

Algorithm for the Averaged MHD Equations*

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A "reduced" set of helically symmetric MHD equations has been previously derived and employed in numerical studies of ideal, "bubble-like" plasma deformations. If resistivity is included, however, the equations must be averaged with respect to the MHD time scale in order to eliminate the MHD restriction on the time step and, thus, dispose of the associated restriction to unrealistically large resistivity.

The set of "averaged" MHD equations is

$$\nabla^2 \frac{\partial\psi}{\partial t} = -\frac{\delta J_z}{\delta t} - \frac{\partial J_z}{\partial\psi}\frac{\partial\psi}{\partial t}$$

and

$$\oint \frac{d\ell}{\nabla\psi}\left[\frac{\partial\psi}{\partial t} + \eta\, J_z(\psi)\right] = 0\,.$$

Here, ψ is the helical flux, t is the time, η is the resistivity, J_z is the current, which is a function only of ψ, $\frac{\delta J_z}{\delta t}$ is the rate of change of the function J_z at constant ψ, and $\oint d\ell$ is the line integral around a flux surface. The case where the resistivity is not constant can also be treated. A numerical algorithm for analyzing this simplified set of equations will be discussed.

*Work supported by the U. S. Atomic Energy Commission.

Analytic and Computational Aspects of the Reduced MHD Equations*

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One of us (MNR) has recently derived a set of simplified nonlinear equations to describe the MHD behavior of Tokamaks. This reduced set of equations is obtained by assuming an initial helical perturbation and keeping terms only to lowest significant order in the usual Tokamak expansion.

The numerical solutions of these equations near integral q indicate that, for a constant current profile, the plasma surface undergoes violent bubble-like deformations. However, with the introduction of shear (a parabolic current profile is employed), distortions are greatly suppressed for $m \geq 2$.

Here, we report on an analytic calculation, which is based on a model current profile and shows the nonlinear stabilizing effect of shear. We have also developed a nonorthogonal code in order to treat shear cases with significant distortion. Finally, a method has been devised for modifying these equations so that perhaps the pressure driven Mercier and internal kink modes can be studied. A comparison of the numerical and analytical results will be presented.

*Work supported by the U. S. Atomic Energy Commission.

Excitation of Alfvén Waves by Ion Beams in Tokamaks*

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ABSTRACT

In a tokamak with radially varying q there is a continuum of singular Alfvén modes with $\omega \equiv k_{\|}(r)V_A$ in ideal MHD theory. With the inclusion of finite-ion gyroradius a discrete set of modes occur, and in this order damping due to collisions of trapped electrons occurs.

When a high-energy ion beam is introduced wave destabilization my occur. Even for beam velocity somewhat less than Alfvén speed the existence of magnetic gradient drifts produces higher harmonics of beam circulation frequency and allows wave resonance to occur.

The competition between this and trapped-electron damping determines stability. Modes localized near the center are always unstable and should produce a flattening of beam profile. For TCT parameters however this flattening should only occur over about one-fourth of the radius. Applications to reactor alphas will also be considered.

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Transport Computations for Tokamaks Including Impurity Effects*

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On the basis of transport-code computations, this paper discusses two distinct aspects of present-day and larger tokamaks. The plasma transport model that we assume is a multi-regime model, based on microinstability theory; the model has been extended in an ad hoc way to allow for enhanced transport due to MHD modes when $q(r) < 1$.

We show that neutral gas and impurity influx create a strong tendency for the temperature and current profiles in all present-day ohmically heated tokamaks to shrink to the point at which $q(0) \leq 1$. At this point, the enhanced MHD transport impedes further shrinking, and creates flat temperature and current profiles over the (usually sizeable) region where $q(r) < 1$. This model gives remarkably good agreement with typical results from ST, ATC, and TFR. For higher-temperature tokamaks of the future (for example neutral-beam heated PLT or TCT), our model predicts that the rate of contraction will be much reduced, so that the emergence of a $q(r) < 1$ region can apparently be avoided.

The control of impurities in future large tokamaks by means of a cold plasma blanket is considered. A programmed gas pulse transiently creates a high-density cold region at the plasma edge, such that the density profile is inverted. This absorbs the energy outflux from the main plasma, lowers the charge-exchange neutral energy, and creates an exterior accumulation point for neoclassical diffusing impurities.

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Relativistic Beam Driven Parametric Instability

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The nonlinear stage of the two-stream instability driven by an initially cold highly relativistic electron beam $\left(S = \beta_0^2 \gamma_0 \left(\frac{n_b}{2n_0}\right)^{1/3} > 1;\ \varepsilon = \left(\frac{n_b}{2n_0}\right)^{1/3} \gamma_0^{-1} \ll 1\right)$ is investigated taking into account the coupling with a back-scattered high frequency mode and an ion density perturbation. For this reason the single mode model of O'Neil et al.[1] and Matsiborko et al.[2], in which beam particles are followed numerically, while the plasma is treated using linearized fluid theory, has been extended to include both relativistic and parametric effects. Special emphasis has been put on the question as to how much energy can be extracted from the beam by the two-stream mechanism. It is found that the second strongly enhanced peak of the beam mode (Lampe and Sprangle[3]) plays an important role in optimizing the energy transfer, allowing values up to 20%. Most of the energy goes into the high frequency modes and with it into the electrons in accordance with numerical simulations of Thode and Sudan[4]. In a later stage the ion density perturbation becomes an absolute growing mode, but with algebraic (t^n) rather than exponential growth.

[1]T.M. O'Neil, I.H. Winfrey and J.H. Malmberg (1971) Phys. Fluids 14, 1204.

[2]N.G. Matsiborko, I.N. Onishchenko, V.D. Shapiro and V.I. Shevchenko (1972) Plasma Phys. 14, 591.

[3]M. Lampe and P. Sprangle (1974) Naval Research Laboratory Report.

[4]L.E. Thode and R.N. Sudan (1974) LA-UR-74-1949.

*On leave from Max-Planck-Institut für Astrophysik, Munich, W. Germany.
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Abstract for Sherwood Theory Meeting 1975

Instability of Envelope Solitons

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Large amplitude plasma waves are known to be unstable against modulational instabilities. Some models of turbulence, supported by one dimensional computer simulation, assume a final state as a set of envelope solitons. Recent two dimensional computer solutions show, however, that envelope solitons are themselves unstable against two dimensional perturbations.[1]

Here we prove analytically that envelope solitons are indeed unstable against modulation in the direction perpendicular to their propagation. Marginal stability exists only for $k_\perp = 0$. The breakup of envelope solitons is accompanied by the radiation of Langmuir waves. For very large amplitude envelope solitons the instability growth rates have been analyzed and found to be in good qualitative agreement with the computer results of Ref. 1.

It can also be shown that a monochromatic large amplitude Langmuir wave is subject to a filamentation instability. One expects therefore that already during the formation of solitons the process of perpendicular breakup is in progress and continues after the soliton is formed. In this sense the breakup of envelope solitons is just a continuation of the filamentation instability.

1., J. Denavit, N. R. Pereira, R. N. Sudan, Phys. Rev. Letters 33, 1435 (1974).

Dynamic Feedback For Multimode Plasma Instabilities

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It has been shown that most plasma instabilities of discrete spectrum are completely "observable" and "controllable" with a single pair of suitably located sensor and suppressor. This indicates the existence of a multimode stabilizing feedback whose formulation and design is sought. It is shown that a suitable dynamic or frequency dependent feedback can stabilize all modes of a plasma instability unlike the widely used constant feedback. Methods are developed in which such a feedback structure can be chosen in terms of its poles and zeroes in relation to the same of the plasma transfer function in the complex frequency plane. The synthesis procedure of such a feedback structure in form of an integrated electronic circuit is also discussed. As an example a dynamic feedback for multimode stabilization of a collisional drift wave instability is developed in detail.

WAVELENGTH DISTRIBUTIONS IN GUIDING - CENTER ELECTROSTATIC TURBULENCE

C. E. Seyler, Jr.; Yehuda Salu; David Montgomery;

and Georg Knorr, University of Iowa

Understanding of the collective dynamics of a plasma is often codified in terms of the stable or unstable roots of a linear dispersion relation. Two-dimensional electrostatic strong turbulence at low frequencies ($\omega < \omega_{ci}$) in the guiding center limit is an interesting phenomenon in that there is no hint of any of the interesting effects in any linear dispersion relation or normal mode. Once this point is conceded, a large body of exploitable techniques from the theory of Navier-Stokes turbulence become available, and go well beyond existing plasma "strong turbulence" theory in precision and sophistication. Here we test numerically the relaxation to the two-temperature canonical distribution of R. H. Kraichnan.[2,3] For a continuous inviscid Navier-Stokes fluid (or guiding center plasma) in two dimensions, energy and "enstrophy" (mean square charge density) are constants of the motion which survive a truncation in Fourier space and which act as independent isolating integrals of the (apparently ergodic) motion. A canonical ensemble based upon these two constants of the motion predicts an equilibrium electric field spectrum

$$< | \underset{\sim}{E} (\underset{\sim}{k}) |^2 > = (\alpha + \beta k^2)^{-1}$$

where α^{-1}, β^{-1} are the two "temperatures", either one of which (but not both) may be negative, depending upon initial values of energy and enstrophy. Approach to this equilibrium has been followed numerically, and several predictions of the Kraichnan ensemble have been confirmed to a striking degree.[1] An inviscid Eulerian Direct Interaction description[1,4] has been derived which presumably describes the details of this approach to equilibrium (this has yet to be verified). The exact relation of the continuum description of the guiding center plasma to the Lin-Onsager discrete vortex model (which leads to a canonical ensemble with a single temperature[5]) remains a puzzle.

1 University of Iowa Report 75-4 (January, 1975).

2 Phys. Fluids 10, 1417 (1967)

3 "Statistical Dynamics of Two-Dimensional Flow" (submitted to J. Fluid Mech., 1974).

4 see also J. R. Herring et al, J. Fluid Mech. 66, 417 (1974).

5 Phys. Fluids 17, 1139 (1974), Eq. (22).

HYBRID MODEL STUDIES OF MAGNETIC FIELD DIFFUSION STUDIES

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It has been shown[1] that the hybrid model (Vlasov ions-fluid electrons) is useful in modeling macroscopic plasmas because it frees one from following the plasma on the timescales of ω_{pe} and lengthscales of λ_D, while allowing for the effects of reflected ion beams and large ion gyro-radii in a natural way. The effect of the microscopic electron dynamics is included on the average by specification of (anomolous) resistivity and thermal conductivity coefficients. Chodura[2] has proposed that the simple resistivity algorithm $\eta = m_e\nu/ne^2$, $\nu = c\,\omega_{pi}(r)\,[1 - \exp\{|v_d(r)|/fv_s(r)\}]$ satisfactorily describes the behavior of the Garching theta pinch. Here $v_d(r)$ is the electron drift velocity, $v_s(r)$ is the ion sound speed, $c = 0.5$, $f = 5.0$. We have generalized this hybrid model to include all field components and an anisotropic resistivity, and have applied it to the ZT-1 screwpinch experiment at Los Alamos. This experiment has initial toroidal bias fields of 600 to 2000 gauss and peak driving poloidal fields of 1000 to 2800 gauss at the wall, with a rise time of $\sim 2\mu s$. In particular, we find that the resistivity

$$ne^2\eta_{\parallel}/m_e = \nu_{\parallel} = \max\left(c_{\parallel}\omega_{pi}(r)\{1-\exp[|v_{d\parallel}(r)|/f_{\parallel}\, v_s(r)]\}, \nu_{fp}\right)$$

$$ne^2\eta_{\perp}/m_e = \nu_{\perp} = \max\left(c_{\perp}\omega_{pi}(r)\{1-\exp[|v_{d\perp}(r)|/f_{\perp}\, v_s(r)]\}, \nu_{\parallel}\right)$$

where $\parallel$, $\perp$ refer to the local direction of the magnetic field, $c_{\parallel} = c_{\perp} = 1.0$, $f_{\parallel} = f_{\perp} = 3.0$ and ν_{fp} is the resistivity given by eg., Spitzer[3] adequately describes the evolution of the magnetic field profiles with time, while $\nu_{\parallel} = \frac{1}{2}\nu_{\perp} = \nu_{fp}$ does not. Specifically, we find considerable diffusion of the driving poloidal field while at the same time we find the toroidal bias field to be compressed and to not diffuse away. The c and f cannot be varied by more than a factor of two without destroying the good agreement. An additional effect being studied with this code is energy loss by u.v. line radiation from impurity ions (whose ionization state populations are determined by the coronal equilibrium model). In the higher density regimes, this loss will be energetically significant on the implosion timescales if the fractional abundance is 1%, and may be important at even lower levels. The principle effect of the impurities is seen when $T_e \sim 5$ to 20 eV, because the cooling rate has a sharp peak in this regime. We illustrate the effect of 1% oxygen impurity on the ZT-1 implosion.

1 IAEA meeting, Tokyo 1974.

2 Biskamp, D., Chodura, R., Dum, C., Proc. 6th European Conf. on Controlled Fusion and Plasma Physics, (Novosibirsk, 1973), Vol. 1, p. 461.

3 Spitzer, L. Jr., Physics of Fully Ionized Gases, Interscience, New York, 1960.

NUMERICAL CALCULATIONS OF INTENSE ION BEAM PRODUCTION

OF THERMONUCLEAR MICROEXPLOSIONS*

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A Monte Carlo numerical computation code has been used to calculate the compression and heating of a solid sphere of deuterium-tritium by a spherically symmetric incident beam of deuterons. "Scientific breakeven" was exceeded for shaped 25 nanosecond pulses of 0.5 meV deuterons in the 300-900 kilojoule range which were incident on a 1 mm radius DT sphere. A simple approximate analysis was developed for the ion heating wave which drives the implosion of the sphere. The necessary power and energy are similar to the requirements for heating by lasers and relativistic electron beams. Recent developments in collective ion acceleration, plasma focus production, and the reflex triode ion source offer hopes for eventually meeting these requirements.

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Enhanced Transport in Non-Axisymmetric Toroidal Devices due to Converstion of Trapped to Circulating Particles by Fluctuations*

by

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Non-axysymmetric toroidal devices such as stellarators contain three classes of particles: 1)circulating, 2)particles trapped in mirrors of the helical field, and 3) particles trapped in mirrors of the toroidal field. Circulating particles tend to be contained inside magnetic surfaces, but trapped particles drift across the magnetic surfaces and, instead, are confined on drift surfaces which are bounded by mod-B surfaces.[1] In a stellarator, the mod-B surfaces deviate considerably from the flux surfaces, and in fact, pass across the boundary between closed and open flux surfaces, i.e., the separatrix.

Hence, if some process converts a trapped particle to a circulating particle while it is outside of the separatrix, that particle will be lost. Normally, as long as the action is an adiabatic invariant, this will not happen, even if there are temporal and spatial variations in the fields.

Furth and Rosenbluth[2] developed a theory of enhanced transport for non-axisymmetric devices in which low-frequency electrostatic potentials caused trapping of circulating particles, or additional trapping of trapped particles to produce the additional transport. It is proposed here that the inverse process, that is, "untrapping" of trapped particles due to electrostatic or magnetic fluctuations may also produce enhanced transport in non-axisymmetric devices. The fluctuations, however, must add energy to the trapped particles in the direction parallel to the d.c. magnetic field, so that they will be untrapped.

A recent experiment on the Proto-Cleo stellarator[3] provided some verification of this mechanism. Enhanced diffusion was observed when transit-time magnetic pumping was attempted. In this experiment, the trapped particle bounce frequency was approximately equal to the TTMP driving frequency, and thus a convenient mechanism for transferring energy to the trapped particle existed, and J was no longer conserved. Thus, low-frequency fluctuations, usually present in stellarator discharges, if they are near the bounce frequency should result in a similar enhanced transport.

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1. S. Fisher, H. Grad, Y. Sone and J. Staples, Plasma Physics & Controlled Nuclear Fusion Research, IAEA, Vienna (1969), p. 899, v. II
2. H.P. Furth and M.N. Rosenbluth, Plasma Physics & Controlled Nuclear Fusion Research, IAEA, Vienna (1969), p. 821 v. II
3. D.J. Lees, W. Millar, et. al., Proc. 5th European Conference on Controlled Fusion & Plasma Physics, Grenoble (1972), v. II, p. 135.

Poloidal Flux in Stellarators*

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Questions have often been raised as to whether a net poloidal flux exists in stellarators and other non-axisymmetric devices, since often no circulating current is provided to produce the rotational transform. Thus

$$\oint \vec{B}_p \cdot d\vec{\ell} \equiv 0 \qquad (1)$$

where the path of integration encloses the magnetic axis and is inside the helical windings. B_p is the poloidal magnetic field. If the path of integration is made to follow along a magnetic surface, it is not possible to obtain the poloidal field from equation (1), since B_p varies along the path. The poloidal flux, however, does not satisfy equation (1).

Accordingly, a numerical evaluation of the poloidal flux was made using the TORFLD[1] code. The stellarator fields utilized for this purpose were those of the Proto-Cleo[2] ℓ =3, 7 field period configuration, which exhibits medium shear. The poloidal flux was computed through a ribbon bounded between two field lines that traced out two closely spaced nested flux surfaces well inside the separatrix and fairly close to the magnetic axis.

The non-axisymmetries due to the helical windings requires that the distance between two field lines does not monotonically increase, as would be the case for a tokamak with shear, but instead, the distance between the field lines alternately increases and decreases in traversing a field period. Thus, the poloidal fields, although they reverse directions in going along a field line, can still produce a non-zero poloidal flux, because of the variable distance between the flux surfaces.

*This work was supported by the National Science Foundation under grant ENG-7203778 A01

1. A. Gibson and D.W. Mason, Plasma Physics, 11, 121 (1969)
   T.J. Martin, Culham Report No. CLM-PDN 2/73 (1973)

2. R.A.E. Bolton, et. al., Phys. Fluids, 14 1566 (1971)

ABSTRACT

INSTABILITIES IN ROTATING THETA PINCHES

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Since incompressible ideal MHD motions about rigidly rotating confined theta pinch equilibria of infinite length are unstable, it is important to determine the spectrum of growth rates and frequencies of gross eigenmodes. Due to the non-selfadjointness of this problem, the intuition gained from the exhaustively studied corresponding static screw pinch problem does not apply, and attempts to solve this problem numerically have led to puzzles whose resolution requires new analytical investigations.

In the present contribution, a complete qualitative picture of the distribution of the eigenvalues in the complex plane is obtained: Continua and accumulation points are determined, monotonicity properties of eigenvalues in their dependence on mode numbers and equilibrium parameters are established, and those regions of the complex plane are identified in which eigenvalues may be present. It is found that unstable eigenvalues can emerge from any points of the real axis as the equilibrium is varied, thus vitiating any standard kind of marginal analysis. Eigenmodes with small mode numbers (gross modes) may be stable for both large and small values of the distance between the rotation axis and the wall, but unstable for intermediate values, thus refuting the belief that moving the wall closer should always reduce growth rates. This analysis will serve as a foundation for numerical investigations.

3-D Magnetohydrodynamics of Noncircular Tokamaks

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Rosenbluth's nonlinear, approximate tokamak equations of motion are generalized to three dimensions. The equations describe MHD in the low β, incompressible, large aspect ratio limit. The equations are solved numerically to study kink modes in tokamaks with rectangular cross section. The rectangular shape is for computational convenience. We consider fixed-boundary kink modes, for which the plasma completely fills the conducting chamber. These modes, which are marginally stable to lowest order in circular tokamaks, become unstable with large growth rates, comparable to the growth rates of free boundary kink modes. The unstable modes are found using a linearized, two-dimensional version of the equation. The linear results are used as initial values in the nonlinear, three-dimensional computations. The nonlinear results show that the magnetic field is perturbed only slightly, while a large amount of plasma convection takes place carrying plasma from the center of the chamber to the walls.

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A CODE FOR CALCULATION OF SYNCHROTRON RADIATION IN TOKAMAK PLASMAS

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The SNCTR code is being developed as a tool for calculation of energy transport by synchrotron radiation which is expected to be important in large scale low β reactors. The purpose of the code is to enable calculation of energy transport and losses for realistic geometries and including the full complexity of the frequency and angular dependence of the interaction between plasma and radiation. Basically the code operates by blanketing the system by a random assortment of rays and then integrating the equation of transfer along these rays for randomly chosen frequencies while calculating the energy gained or lost by each zone traversed. The code has options for including specular or diffuse wall reflection with arbitrary reflection coefficients, and for including two polarizations with arbitrary mixing between them upon reflection. Two modes of operation are possible in which either the zone by zone heating rates are calculated, or the emergent radiation spectrum at any point on the boundary is generated. At present the code uses a low temperature opacity model in which line broadening is due to field inhomogeneities alone. A more realistic model will soon be incorporated.

Trapped-Particle Instabilities: Destabilization from Curvature Drift Resonances and Stabilization from Inverted Gradient Profiles*

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This paper treats two new effects which are found to strongly influence the trapped-particle stability picture in tokamaks. First it is shown that electron toroidal gradient drift resonances, ignored in earlier work,[1,2] can make an important destabilizing contribution to the trapped-electron modes. Secondly, it is found that for inverted density profiles, which can occur under certain modes of operation in TCT (Two-Component Torus), the stability against both trapped-electron and trapped-ion modes is greatly improved.

With regard to the first topic, previously considered effects arising from collisions, temperature gradients, shear,[1] and finite ion gyroradius[2] are all included with the curvature drifts in the analysis, and the resultant eigenvalue equation is solved numerically without commonly used perturbation approximations.[1,2] For typical tokamak parameters the maximum growth rates are found to be increased over earlier estimates[1,2] by roughly a factor of 4, and requirements on magnetic shear strength for stabilization are likewise more severe and very difficult to satisfy.

In the case of inverted density profiles it is found that the trapped-ion mode can be easily stabilized. The calculations indicate that there is a marked reduction in the collisional growth together with an enhancement of all previously considered damping effects.[3] For the trapped-electron modes the inverted profile renders ineffective the new destabilizing mechanism described above. As a result these modes can be stabilized for achievable values of shear provided the temperature gradients are not too severe.

[1] W. Horton, Jr., et al., 5th Conf. on Plasma Phys. and Contr. Nucl. Fusion Res., Tokyo, Japan, 1974, paper IAEA-CN-33/A14-3.

[2] C. S. Liu, M. N. Rosenbluth, and W. M. Tang, PPPL Report MATT-1125 (1974).

[3] W. M. Tang, Phys. Fluids 17, 1249 (1974).

* Work supported by U.S.E.R.D.A. Contract AT(11-1)-3073.

Abstract for CTR Theory Meeting, April 7-9, 1975

INITIAL SHOCK HEATING OF A BELT PINCH

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Magnetic field compression of an elongated, toroidal belt pinch is simulated with a two-temperature, one-fluid MHD code which includes the complete, anisotropic classical transport coefficients for electrical and thermal conductivity and viscosity. Shocks propagate radially inward from the pinch container walls and bounce several times near the center as the rising magnetic fields pinch the shock-heated plasma. The Lagrangian, 1-D (r only) MHD code is coupled to the external circuit equations to account for the time-dependent inductance. For quarter-cycle magnetic field maxima of $B_\phi = 18$ KG, $B_\theta = -1$ KG at the inner radius (15 cm.) and $B_\phi = 9$ KG, $B_\theta = 2.3$ KG at the outer radius (30 cm.), the computed electron and ion temperatures are on the order of 20 ev for an initial hydrogen fill density of 10^{15} cm.$^{-3}$. The toroidal shift after each column implosion is observed. Computed results are compared to recent experimental data.

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Abstract Offered for the 1975 Annual Meeting on
Theoretical Aspects of Controlled Thermonuclear Research

Dissipative Trapped-Electron Instabilities in the Transition from the Banana to Plateau Regime*

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Of the various types of low frequency drift and trapped-particle instabilities,[1] the most relevant mode in present tokamaks is the drift-dissipative trapped-electron instability. This instability has been derived only in the "banana" regime where the trapped electrons are very collisionless. However, present experiments tend to operate in the transition regime between the banana and plateau regimes ($\nu_{*e} \sim 1$).

In order to investigate the dissipative trapped-electron instabilities in this transition, we separate velocity space into various regions of collisionality (banana, plateau, boundary layer), calculate the perturbed distribution functions accordingly, and appropriately average over velocity space to obtain the perturbed density. This phenomenological procedure has been found to be very useful in analytically reproducing[2] the neoclassical diffusion coefficient in the transition from the banana to plateau regime,[3] which has previously been computed only numerically, and in elucidating the physics involved in this transition. For the temperature-gradient-driven dissipative trapped-electron modes[1] we find that as ν_{*e} increases the usual destabilizing collisional growth integral does not change sign; however, it does become exponentially small for $\nu_{*e}^{1/2} > 1$. For $\nu_{*e} \gtrsim 1$ a collisional broadened Landau-type resonance, which for $\omega \simeq \omega_*$ with a temperature gradient has a stabilizing effect similar to that in collisionless drift modes,[4] tends to stabilize the mode. The effects of boundary layers, finite ion gyroradii, poloidal, and radial mode structures are also included in the calculation. In general we find that for $k\rho_i \ll 1$ the shear stabilization criterion is relaxed slightly for $\nu_{*e} \lesssim 1$ and that the temperature gradient driven modes can be stabilized completely for $\nu_{*e} \gtrsim \alpha$, where $\alpha \sim 1$.

[1] B. B. Kadomtsev and O. P. Pogutse, Nuclear Fusion 11, 67 (1971).
[2] K. T. Tsang and J. D. Callen, "On the Smooth Transition of Neoclassical Diffusion from the Banana to Pfirsch-Schlüter Regime," ORNL-TM-4848, Feb. 1975.
[3] F. L. Hinton and M. N. Rosenbluth, Phys. Fluids 16, 836 (1973).
[4] See for example: J. N. Davidson and T. Kammash, Nuclear Fusion 8, 203 (1968).

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USE OF IMPLODING LINERS IN STRONG ADIABATIC COMPRESSION OF BEAM-TARGET FUSION PLASMAS

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Major- and minor-radius adiabatic compressions, which have been demonstrated[1] to be effective auxiliary heating mechanisms in a tokomak plasma, are limited to increasing temperatures by a factor ~ 2. Compression of axial magnetic flux using an imploded cylindrical liner, in contrast, can produce tenfold reductions in linear scale, implying energy increases by a factor ~ 100. Theoretical calculations have been performed for fusion systems in which ion rings are compressed along with target plasmas to achieve proper conditions for beam-plasma fusion. Unlike the scheme of Sudan and Ott[2], the ring ions produce nearly all the fusion energy through collisions with the target ions, rather than merely providing a closed field configuration for a thermonuclear plasma. With only modest target plasma temperatures, simple scaling calculations indicate that values of $Q > 1$ can be obtained for advanced reaction systems like $D\text{-}He^3$, thus allowing direct conversion of nuclear energy to electricity in a relatively clean radiation environment. Numerical computations, including beam-plasma scattering, liner dynamical motion and nuclear energy deposition are used to study such systems in detail.

1. K. Bol, et. al., Phys. Rev. Lett. 32, 661 (1974); D. L. Jassby and H. P. Furth, Princeton Univ. Rept. MATT-1048 (1974).

2. R. N. Sudan and Edward Ott, Phys. Rev. Lett. 29, 1495 (1972).

Resistive Instabilities at a Divertor or Poloidal Field X-Point

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Resistive instabilities in the neighborhood of a magnetic field x-point for divertor or multipole machines are studied. Such instabilities are important when impurities are present. A new ordering for tearing and interchange modes with axially symmetrical perturbations have been found. Instabilities are shown to be driven by current gradients and the stability criterion is obtained from outer-solutions, as in the standard analysis. The growth rate for tearing modes ranges from $\eta^{5/7}$ to $\eta^{3/5}$ as the angle of the separatrix at the x-point is reduced from $\pi/2$ to 0 and the classical sheet pinch tearing mode is recovered in the small angle limit. The structure of the resistive layer is correspondingly changed. However, the flow velocity of disturbances is of the same order of magnitude as the classical tearing mode. Solutions for interchange modes must be investigated numerically. These solutions satisfy matching conditions from which growth rates can be obtained. As in the case of the tearing mode, the x-point geometry modifies the growth rate.

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A Principle of Detailed Balancing in Tokamak Diffusion and Impurity Ion Screening

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The mean radial diffusion (Γ_j) of any ion species j in a large aspect ratio tokamak is shown to consist of four parts, namely, classical diffusion (Γ_{jc}), neoclassical diffusion ($\Gamma_{jNC} \equiv [P_{j\parallel} - P_{j\perp}] \sin\theta / R_o eB$), a generalized Pfirsch-Schluter type diffusion ($\Gamma_{jPS} \equiv r\tilde{F}_{j\parallel c} / R_o eB_{\theta o}$) and an electric diffusion ($\Gamma_{jE} \equiv \tilde{n}_j \tilde{E}_\theta / B_o$), where $F_{j\parallel}$ is the friction force parallel to $\vec{B}$ experienced by the j-species and where we have taken $F_{j\parallel} = \overline{F}_{j\parallel} + \tilde{F}_{j\parallel}(\theta) = \overline{F}_{j\parallel} + \tilde{F}_{j\parallel s} \sin\theta + F_{j\parallel c} \cos\theta + \ldots$ (From the work in an accompanying paper it follows that for present impure tokamak plasmas the three components other than Γ_c can have comparable magnitudes.) It is well known that to second order in the Larmor radius, there is ambipolarity so that $\Gamma_e = \sum_{ions} Z\Gamma_Z$ independent of the value of E_r. In this paper we show that this ambipolarity applies to each component of the diffusion separately. $\Gamma_{eC} = \Sigma Z\Gamma_{ZC}$, $\Gamma_{eNC} = \Sigma Z\Gamma_{ZNC}$, etc. This principle has important implications for plasmas where the component species are in different collisional regimes, (banana, plateau or Pfirsch-Schluter). In particular in present impure tokamak plasma, where the hydrogen ions (i) are in the plateau regime and the impurities (Z) are well into the Pfirsch-Schluter regime, there can be no fast ion diffusion of the neoclassical type since $\overline{(P_{Z\parallel} - P_{Z\perp})\sin\theta}$ and hence Γ_{ZNC} will be small under these conditions; $\Gamma_{iNC} \simeq \Gamma_{eNC}$. The dominant remaining fast ion diffusion will generally be $\Gamma_{i_E} \simeq -\Sigma Z\Gamma_{ZE}$. For existing Tokamaks this component will usually be negative for the ions, positive for the impurities, i.e., impurity ion screening is predicted.

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Neoclassical Heat Transfer between Electrons and Ions

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The heat equation for electrons (e.g., Braginskii) can be written in the form

$$\frac{3}{2}\frac{dp_e}{dt} + \frac{5}{2} p_e \vec{\nabla}\cdot\vec{v}_e + \vec{\nabla}\cdot\vec{q}_e + Q_{ei} = -\pi_{e_{\alpha\beta}} \frac{\partial v_{e\alpha}}{\partial x_\beta} - \frac{\vec{F}_e\cdot\vec{j}}{ne} \qquad (1)$$

where π_e is the non-hydrostatic part of the electron pressure tensor, $\vec{v}_e$ is the mean electron velocity, Q_{ei} the collisional transfer of heat from electrons to ions and $\vec{F}_e$ is the friction force experience by the electrons. To second order in the Larmor radius only diagonal terms appear in π_e and the first term on the right reduces to $(P_{e_\parallel} - P_{e_\perp})v_{e\theta}\sin\theta$. Hence, taking the volume average between adjacent magnetic surfaces for a tokamak plasma,

$$\left(\frac{3}{2}\frac{\partial\bar{p}_e}{\partial t} + \frac{3}{2}\frac{\Gamma}{\bar{n}}\frac{\partial\bar{p}_e}{\partial r} + \frac{5\bar{p}_e}{2r}\frac{\partial(r\Gamma/\bar{n})}{\partial r} + \frac{1}{r}\frac{\partial(rq_{er})}{\partial r} - Q_{ei} \equiv F_e\right) =$$

$$\Gamma_{NC}\, e\, \bar{V}_\theta B_o - (\Gamma_{NC}+\Gamma_{PS})\frac{1}{\bar{n}}\frac{\partial p}{\partial r} - \left[\frac{\Gamma_E}{\bar{n}}\frac{\partial\bar{p}_e}{\partial r} - \overline{\frac{\tilde{E}_\theta}{B_o}\frac{\partial\tilde{p}_e}{\partial r}}\right] + j_{\parallel_o}E_{\phi o} \qquad (2)$$

Similarly,

$$F_i = -\Gamma_{NC}\, e\, \bar{V}_\theta B_o - \left[\frac{\Gamma_E}{\bar{n}}\frac{\partial\bar{p}_i}{\partial r} - \overline{\frac{\tilde{E}_\theta}{B_o}\frac{\partial\tilde{p}_i}{\partial r}}\right] \qquad (3)$$

where $\vec{\bar{V}}$ is the mean plasma velocity, $p_e \equiv \bar{p}_e + \tilde{p}_e(\theta)$, etc., $p = \bar{p}_e + \bar{p}_i$, $\tilde{p}_e = -\tilde{p}_i$. (A pure hydrogen plasma is taken for simplicity.) The terms $\Gamma_{NC} e \bar{V}_\theta B_o$ and $\overline{(\tilde{E}_\theta/B_o)(\partial\tilde{p}_i/\partial r)}$ involve direct transfer of heat between electrons and ions independent of collisions. The former term, the more important one, is not new since the equations are identical with those of Rosenbluth, Hazeltine and Hinton for $\Gamma_{PS} = \Gamma_E = \tilde{E}_\theta = 0$. In their equations this term did not involve a large electron heat loss since it vanished when the pinching ceased $(\Gamma_{NC} = \Gamma \simeq 0)$ and $\bar{V}_\theta$ $(= -T_i'/2eB_\theta$ for plateau regime) was not large. Here it is shown that for present impure plasmas with hot electron cores (a) the term does not vanish when $\Gamma = 0$, since Γ_{NC} is balanced by Γ_E and (b) $\bar{V}_\theta$ is larger than the above formula. The magnitude of the term can be a significant fraction of $j_{\parallel_o}E_{\phi o}$ and can represent an important electron heat loss for present tokamaks. (The Γ's are defined in an accompanying abstract.)

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Neoclassical Instabilities - Explosive Disruptions

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The new neoclassical transport equations reported in an accompanying abstract, which include the effects of the electrostatic potential $\Phi(\theta)$ and the impurity density variation $\tilde{n}_z(\theta)$, are shown to predict two instabilities, an electron instability whose properties qualitatively match the experimentally observed mini- (or pre-) disruption and a faster ion instability which matches the major disruption. Firstly, overlooking the differences between the various numerical coefficients the qualitative form of the current equation is

$$j_\parallel = \sigma_\parallel E_\varphi - C(\lambda_1 + \lambda_2 X_{ec} + \lambda_3 X_{ec}^2)\left(\frac{1}{B_\theta}\frac{\partial p}{\partial r} + \lambda_4 \sigma_\parallel E_\varphi\right)$$

where the λ's are numerical coefficients of order unity which should be matrices, C is the usual plateau regime coefficient and

$$X_{ec} = \mathrm{Re}\,\Phi_c/rT_e = M_z\,[U_P^2 + (U_\parallel - U_P)^2]\ /\ ZT_e$$

It is shown that $U_P\ (\equiv \frac{B_\varphi}{B_\theta}U_\theta - (1/ne\ B_\theta)\cdot\frac{\partial p_i}{\partial r})$ depends on E_φ in the central core of the plasma where $T_e >> T_i$ and that the above equations become linearly unstable to a perturbation in E_φ in this region when X_{ec} exceeds a critical value. For the correct sign of δE_φ the linear growth rate will increase with time because X_{ec} increases. Secondly, if X_{ic} $(\equiv \mathrm{Re}\,\Phi_c/rT_i)$ exceeds a critical value of order unity, the plasma is subject to an explosive ion instability involving θ-rotation and growth rate of order v_{Ti}/R. This instability will cause a large transfer of energy from electrons to ions (as observed for major disruptions by the Australian workers, Plasma Phys. 13, 849, 1971) and a rapid outward diffusion of impurity ions (as observed experimentally by Vershkov and Mirnov, Nucl. Fus. 14, 383, 1974 - a feature distinguishing major- from minor- disruptions).

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Resistivity in a Trapped-Electron Plasma+

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The distribution-function for electrons in a magnetic trapping geometry does not satisfy the bounce-averaged Boltzman Equation when an electric field is present. More precisely, there is a class of untrapped particles whose motion in velocity space is discontinuous on a time scale for which the trapped-electron bounce-time is considered short. These particles are untrapped electrons whose parallel velocity is reduced by the applied electric field. When such an electron crosses the trapping surface, its velocity changes sign, but remains approximately constant in magnitude, in a bounce time. The theoretical implications of these effects are discussed elsewhere at this meeting by one of the authors (Coppi: Slideaway Distribution).

To assess the importance of these effects on plasma resistivity and heating, it is crucial to include an accurate collisional model in the equation for the evolution of the distribution function. For electric fields of interest in experiments, the collision operator is strongly nonlinear, so this was done numerically. A two-dimensional Fokker-Planck model was modified to treat trapped and untrapped particles, and the non-local transport of untrapped particles was included. Particles in the "trapping" region for which $\omega_{bounce} < \omega_{coll}$ were treated as untrapped. An adjustable "wave-damping" coefficient was included to model electron energy loss to ions (Coppi: mechanism discussed elsewhere).

Calculations were performed as a function of electric field strength and damping rate, to examine their effects on electron resistivity and distribution-function shape. Results included: (1) for moderate electric fields, the resistivity can be much less than the Spitzer value, without a significant runaway population; (2) both the trapped and untrapped electrons can be driven unstable ($\partial f/\partial |v_\parallel| > 0$); and (3) if such an instability takes place in the trapped electrons, it would have low parallel phase velocity, and might be able to resonate with ions, when the frequency of such an instability is below the ion cyclotron frequency.

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INTERACTION BETWEEN INSTABILITIES IN PLASMA HEATING BY RELATIVISTIC ELECTRON BEAMS

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Recent experimental results show[1] that rapid plasma heating up to several keV has been achieved by injecting intense relativistic electron beams ($n_b \simeq 2\text{x}10^{12}$ cm^{-3}, $\gamma \simeq 3$, $t_b \simeq 10^{-7}$ sec) into high density background ($n \sim 10^{15-16}$ cm^{-3}) of neutral or weakly preionized gas. The heating can be largely accounted for by an anomalously low conductivity ($\sigma < 10^4$ mho/m) which indicates the presence of turbulent waves interacting with the backstream plasma electrons. Two types of instabilities may exist in such beam-plasma systems, a low frequency instability ($\omega \lesssim \omega_{pi}$, $k \simeq C_s$) due to the plasma electrons interacting with the ions and a high frequency instability ($\omega \simeq \omega_{pe}$, $k \simeq \omega_{pe}/v_b$) due to the beam electrons interacting with the plasma electrons. We have studied the high frequency instability (e-e) in the presence of finite-amplitude low frequency waves. The low frequency waves can be either linearly excited by the e-i instability or parametrically driven by the e-e waves. The obtained dispersion relation gives a critical amplitude of the low frequency wave above which the e-e instability becomes nonlinearly stabilized. The anomalous resitivity infered from the critical wave amplitude provides a dc ohmic heating for the plasma and gives a stopping length over which a beam electron loses all its forward kinetic energy. Because of the parametric instability driven by the e-e waves, such ohmic heating may even persist after the e-e instability becomes linearly stabilized and hence provide bulk of the plasma heating. The theoretical results compare favorably with experiments. Implications of this nonlinear process on e-beam heating of a high-density ($n_e \sim 10^{16-17}$ cm^{-3}) linear reactor will also be discussed.

[1] Prono, D., B. Ecker, N. Bergstrom, J. Benford, and S. Putnam, "Plasma Heating with Ultra-Intense Relativistic Electron Beams," Physics International Report PIFR-557, Feb. 1975.

Nonlinear Rotational Stability of Tokamak MHD Equilibria with Inertial and Viscous Effects

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A self-consistent (finite-β) analysis of the rotational spin-up, first discussed in Ref. 1, of resistive MHD toroidal equilibria has been carried out including both inertial and viscous effects. A tokamak expansion is employed; nevertheless, resistivity and viscosity enter as free parameters in our treatment. The first important result we find is that inclusion of inertia vitally affects both the form and magnitude of the viscous stresses. With the inclusion of inertia, both stable and unstable initial states can be found, depending on the nature and magnitude of the initial toroidal rotation. The influence of viscosity, however, is invariably stabilizing; at sufficiently high temperatures it appears that the rotation will always be damped toward a completely determined steady state which is itself affected by viscosity, as well as by inertia. These phenomena are also significant in determining the radial diffusion (mass flux).

1. R. D. Hazeltine, E. P. Lee, M. N. Rosenbluth, Phys. Rev. Lett. 25, 7, 427 (1970).

Where

1. $\frac{\partial}{\partial z} = \frac{k}{m}\frac{\partial}{\partial \theta} \Rightarrow 2D$

2. $B_z^2 \gg B_\theta^2$, lowest order in kr

$$\nabla_\perp \cdot V = 0$$

$$\frac{\partial \psi}{\partial t} + V \cdot \nabla \psi = -\eta \nabla^2 \psi$$

$$\rho \frac{dv}{dt} = \nabla \left\{ -(p + \frac{B^2}{2}) + \frac{\nabla \psi^2}{2} - 2kB_z \right.$$

$\eta \neq 0$

$\frac{\partial}{\partial t} \sim V \sim \eta \Rightarrow \frac{d}{dt} \nabla^2 A \sim \eta^2$

$\frac{\partial \eta}{\partial t} + V \cdot \nabla \eta = D B \cdot \nabla (\frac{B}{B^2} \nabla \eta)$

$\eta = \eta_0 (\psi)$

$\therefore \nabla \psi \times \nabla (\nabla^2 \psi) = 0$

$\Rightarrow \nabla^2 = f(\psi) \quad \forall t$

$\nabla^2 \psi = -J(\psi) - \frac{2kB_z}{m}$ sequence of equil MHD states

$\nabla^2 \dot\psi = -\frac{\partial J}{\partial \psi} \dot\psi - \frac{\partial J}{\partial t}$

$\frac{\partial \eta_0}{\partial t} + \frac{\partial \eta_0}{\partial \psi} \dot\psi + \cdots$

$\int \frac{dl}{\nabla \psi} (V \cdot \nabla \psi) = 0 \Rightarrow$

$\oint \frac{dl}{\nabla \psi} \left[\frac{\partial \psi}{\partial t} + \eta J_z \right] = 0$

$t \to \frac{\tau_R}{\tau_H} t, \quad \tau_R = \frac{a^2}{\eta^2}$

$\tau_H = \tau_A \; (?)$